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NUCLEI
Theory

Increase in the Probability of Forbidden Electron Beta Decays in a Superstrong Magnetic Field

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Abstract—Form factors for unique forbidden electron beta decays in a superstrong constant uniform external magnetic field are considered. The probability of forbidden and allowed electron beta decays increases in a superstrong magnetic field owing to the increase in the density of vacant electron bound states at the nucleus involved. It is shown that, because of the growth of the form factors, the relative increase in the probability of forbidden electron beta decays in a magnetic field exceeds the relative increase in the probability of allowed decays (at identical decay endpoint energies).

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1. INTRODUCTION

Nuclear processes occurring in the presence of a superstrong magnetic field, $H \gg \alpha^2 H_0 \sim 2.35 \times 10^9$ G (where α is the fine-structure constant; $H_0 = c^3 m_e^2 e^{-1} \hbar^{-1} \sim 4.41 \times 10^{13}$ G; c is the speed of light; m_e and e are the electron mass and charge, respectively; and $\hbar$ is the Planck constant), are of interest in connection with problems that arise in studying neutron stars and their dynamics [1–5]. A magnetic field of strength $H \sim \alpha^2 H_0$ corresponds to the equality of the electron Larmor radius to the Bohr radius; at $H \sim H_0$, the Larmor radius is on the same order of magnitude as the electron Compton wavelength. It is assumed that magnetic fields of neutron stars are as high as about 10^{13} G. In [6], Kadomtsev showed that a superstrong magnetic field changes the structure of an atom substantially. In response to changes in the atomic electron shell, the nuclear-decay energy also changes, which, in turn, leads to a change in the nuclear-decay probability [7, 8]. The plasma of a neutron-star atmosphere was studied in [2], where it was assumed that the atmosphere of a neutron star and the surface layer of its outer crust (several hundred meters) have a low density (on the neutron-star scale) of about 10^4 g/cm^3 and that the surface plasma is not fully ionized. Therefore, the surface plasma of a neutron star proves to be in a superstrong magnetic field, so that decays to bound states and other processes involving an atomic shell in a superstrong magnetic field, which are considered in the present study, are of importance here. We note that, despite its small size, the surface layer determines the parameters of the radiation escaping from the star.

A relativistic consideration of neutron beta decay on the basis of the Dirac equation in a superstrong magnetic field was performed in [9] with allowance for proton recoil but without allowance for bound electron–positron states. In [10], neutron decay in a superstrong magnetic field, $H > H_0$, was considered by using the Bethe–Salpeter equation to take into account the bound states in question. In so strong a field, electrons occupy only the first Landau level of transverse motion. The opposite case of $H < H_0$ is of no interest for an analysis of neutron beta decay (the beta-decay energy is 782.45 keV in this case), since the probability of neutron decay to a bound proton–electron state is low (about 3×10^{-6} in an unperturbed state at $H = 0$ [11]). This case may be of interest for beta decays characterized by low endpoint energies (for example, tritium beta decay, which has the endpoint energy of 18.61 keV, the probabilities of decay into a bound state of ions and atoms being about 1% and 0.6%, respectively), since the probability of decay to a bound state increases with increasing nuclear charge and with decreasing decay endpoint energy [11, 12]. In [13], positronium bound states in magnetic fields satisfying the condition $H \gg \alpha^2 H_0$ for the ground state of transverse motion were studied on the basis of the Bethe–Salpeter equation. A more detailed consideration of positronium in a magnetic field was performed by Shabad and Usov [14].

In [15], the spectrum of electron bound states in the Coulomb field of a nucleus in the presence of an external magnetic field satisfying the condition $H \gg \alpha^2 H_0$ was determined in the nonrelativistic approximation with the aid of the Schrödinger equation.

In [16, 17], the Dirac equation was used to study electron states in the Coulomb field of a nucleus and an external magnetic field satisfying the condition $H \gg H_0$ that are associated with the first Landau level of transverse motion: the ground state and the spectrum of excited bound states of longitudinal motion were considered in [16] and [17], respectively. The increase in the probability of allowed beta decays of nuclei with allowance for electron bound states in the electric field of a nucleus was calculated in [18] on the basis of the Dirac equation for the case where decay may occur not only to the first Landau level but also to higher Landau levels. An increase in the density of vacant bound states of electrons at a nucleus is the main reason for the change in this probability. The results obtained in [18] are applicable under the condition $H_0 > H \gg \alpha^2 H_0$, since, for $H \geqslant H_0$, it is necessary to take into account quantum corrections to the electron mass and magnetic moment [14, 19].

Not only does the density of vacant bound states increase for forbidden beta decays, but also decay form factors, which depend on the distributions of leptons, change in this case. The probabilities of unique forbidden and allowed beta decays to an electron bound state under conditions of the ionization of an atom were examined in [20]. In the present study, form factors for unique forbidden beta decays in a superstrong magnetic field, $H_0 > H \gg (\alpha Z)^2 H_0$, are determined for nuclei of moderately small charge satisfying the condition $Z \ll \alpha^{-1}$. Such fields are superstrong on an atomic scale, but they are weak in relation to nuclear fields. Their effect on beta decay is indirect: the magnetic field changes bound states of electrons, and this leads to a change in decay to a bound state. Concurrently, it is assumed in the present study that the magnetic field does not change the nuclear components of decay matrix elements.

Under terrestrial conditions, superstrong pulsed magnetic fields are attainable in powerful femtosecond lasers [21]. In a pulse of duration in excess of 100 fs, the energy density at the target position may be as high as 10^{20} W/cm^2. It was experimentally confirmed that magnetic fields of strength $(0.7 \pm 0.1) \times 10^9$ G may be available [22]. Although the time of their action is short (about 10^{-13} s), it is sufficient for the occurrence of atomic processes [23] and for the observation of changes in isomer half-lives [24].

2. MATRIX ELEMENTS OF FORBIDDEN BETA DECAYS

We are now going to find out how nonzero matrix elements of forbidden beta decays depend on the strength of an external magnetic field. In doing this, we assume that the magnetic field changes only electron distributions. The total probability of the beta decay of a nucleus, λ, has the form (here, we make use of the system of relativistic units, where $\hbar = c = m_e = 1$)

$$\lambda = \frac{g^2}{2\pi^3} \sum_l |M(l)|^2, \qquad (1)$$

where g is the weak coupling constant and $M(l)$ is the matrix element for decay to a specific state of leptons that is characterized by the set of quantum numbers l, the sum being taken over all possible lepton states. Within the $V{-}A$ theory of weak interaction, the general expression for M in the approximation of independent nucleons has the form of the sum of five terms; that is, $M(l) = \sum_a C_a M_a(l)$, where a labels the scalar (S), vector (V), tensor (T), axial-vector (A), and pseudoscalar (P) terms; C_a stands for the corresponding coupling constants [25, 26]; and

$$M_a(l) = \int \left[\sum_i \bar{\Psi}' O_{a,i} \tau_i \Psi \right] \times \left[\bar{\psi}_e O_{a,L} \left(1 + \gamma^5\right) \psi_\nu \right] d^3r + \text{h.c.} \qquad (2)$$

Here, Ψ and Ψ' are, respectively, the initial- and the final-state nuclear wave function in the form of the product of spinors of all nucleons; τ_i is an operator that transforms the ith neutron of the nucleus involved into a proton (in the sum, the subscript i runs through all intranuclear nucleons); ψ_e and ψ_ν are spinors that describe the electron and neutrino, respectively; the overbar on $\bar{\psi}$ means Dirac conjugation; $O_S = 1$, $O_V = \gamma^\mu$, $O_T = \gamma^\mu \gamma^\nu$, $O_A = \gamma^\mu \gamma^5$, and $O_P = \gamma^5$, where γ are the Dirac matrices ($\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$) and μ, $\nu = 0, 1, 2, 3$; and h.c. corresponds to Hermitian conjugation. On the matrices O_a, the index i means action on the ith nucleon, while the index L means action on the lepton function. Integration is performed over the volume of the nucleus.

In the following, we describe the nucleus in the nonrelativistic (Pauli) approximation of independent nucleons in the reference frame comoving with the nucleus and disregard the recoil momentum (we place the coordinate origin at the center of the nucleus). With allowance for the fact that the Dirac and the Pauli matrices σ^j are related by the equation

$$\gamma^{j=1,2,3} = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix},$$

the matrix elements in question are simplified to become (the pseudoscalar matrix element vanishes in this approximation)

$$M_S = \int \Re_V(\mathbf{r})\, \bar{\psi}_e \left(1 + \gamma^5\right) \psi_\nu d^3r, \qquad (3)$$

$$M_V = \int \Re_V(\mathbf{r})\, \bar{\psi}_e \gamma^0 \left(1+\gamma^5\right) \psi_\nu d^3 r,$$

$$M_T = \int \Re_{Aj}(\mathbf{r})\, \bar{\psi}_e \gamma^0 \gamma^j \left(1+\gamma^5\right) \psi_\nu d^3 r,$$

$$M_A = \int \Re_{Aj}(\mathbf{r})\, \bar{\psi}_e \gamma^j \left(1+\gamma^5\right) \psi_\nu d^3 r,$$

$$\Re_V(\mathbf{r}) = \sum_i \Psi'^{+}_{\mathrm{P}} \tau_i \Psi_{\mathrm{P}},$$

$$\Re_{Aj}(\mathbf{r}) = \sum_i \Psi'^{+}_{\mathrm{P}} \sigma_{j,i} \tau_i \Psi_{\mathrm{P}},$$

where Ψ_{P} and Ψ'_{P} are, respectively, the initial- and the final-state nuclear wave function in the Pauli approximation; Ψ^+ is the Hermitian conjugate of Ψ; summation is performed over dummy indices; and $\sigma_{j,i}$ acts on the ith nucleon.

The total probability of forbidden beta decay is calculated according to the following scheme: the matrix elements and the probability of the respective transition are determined for specific states of the electron and neutrino, whereupon the resulting probability is summed over all admissible lepton states with allowance for conservation laws. The main problem in calculating the probability of forbidden decays is that the contributions of different orders in the expansion of the various matrix elements in (3) in r, which are determined by specific nuclear functions, may be commensurate in magnitude.

Let us represent the angular dependences of all functions in the integrands in the matrix elements (3) for the decay process under study in the form of an expansion in spherical harmonics [27]

$$Y_l^m(\theta,\varphi) \tag{4}$$

$$= \sqrt{\frac{(l-m)!}{(l+m)!}\frac{(2l+1)}{4\pi}} P_l^m(\cos\theta)\, e^{im\varphi},$$

where $P_l^m(\cos\theta)$ are associated Legendre polynomials. It is well known that spherical harmonics satisfy the relation [27]

$$Y_{l_1}^{m_1}(\theta,\varphi)\, Y_{l_2}^{m_2}(\theta,\varphi) \tag{5}$$

$$= \frac{1}{2\sqrt{\pi}} \sum_{l=l_{\min}}^{l_1+l_2} \tilde{C}^l_{l_1,m_1,l_2,m_2} Y_l^{m_1+m_2}(\theta,\varphi),$$

where

$$l_{\min} = \max\left(|l_1 - l_2|; m_1 + m_2\right). \tag{6}$$

The coefficients $\tilde{C}^l_{l_1,m_1,l_2,m_2}$ are expressed in terms of Clebsch–Gordan coefficients as

$$\tilde{C}^l_{l_1,m_1,l_2,m_2} \tag{7}$$

$$= \sqrt{\frac{(2l_1+1)(2l_2+1)}{(2l+1)}} C^{l,0}_{l_1 0, l_2 0} C^{l,m_1+m_2}_{l_1 m_1, l_2 m_2}.$$

Relevant values of Clebsch–Gordan coefficients are given in [27].

Transitions for which the nucleus-averaged value of at least one nuclear function $\langle \Re_V \rangle$ or $\langle \Re_{Aj} \rangle$ is nonzero—that is, the coefficient of Y_0 in the expansion of the function in the spherical harmonics (4) is nonzero—are allowed beta decays. In such a decay, the parity of the nucleus does not change. If $\Re_V(0) \neq 0$, then the spin of the nucleus does not change either. If $\Re_{Aj}(0) \neq 0$ (at some values of j), the spin of the nucleus changes by unity or remains unchanged (the $0 \to 0$ transition is excluded). Since the characteristic scale of the change in the lepton functions is large in relation to the nuclear size, the values of the lepton functions at the center of the nucleus make a dominant contribution to the probability of allowed beta decays.

For forbidden decays, the nucleus-averaged values of the nuclear wave functions are zero, $\langle \Re_V \rangle = 0$ and $\langle \Re_{Aj} \rangle = 0$, for all values of j. The expansion of the functions $\Re_V(\mathbf{r})$ and $\Re_{Aj}(\mathbf{r})$ for forbidden beta decay in spherical harmonics starts from Y_k; in general, the indices k are different for all functions $\Re$. The smallest of all indices k is referred to as the beta-decay forbiddenness order. The following rules hold for k [25, 26] in terms of the change in the nuclear spin, ΔI, and the change in parity, $\Delta\pi = \pm 1$ (in the case of $+1$, parity remains unchanged): if $\Delta\pi = (-1)^{\Delta I}$, then $k = \Delta I$, but if $\Delta\pi = (-1)^{\Delta I + 1}$, then $k = |\Delta I - 1|$, in which case the respective transitions are unique. This definition of the forbiddenness order covers allowed transitions ($k = 0$). For unique transitions, only one of the tensor (T) matrix elements in (3) is a leading one, the contributions of the others being small. Since spherical harmonics are orthogonal, the expansion of lepton products in a nonzero matrix element must also involve spherical harmonics carrying a subscript larger than or equal to k. Since integration in the matrix elements in (3) is performed over the volume of the nucleus, whose size is assumed to be the smallest parameter in relation to the characteristic scales of changes in lepton functions, it is sufficient, for estimating the matrix elements in (3), to determine the leading term of the expansion of lepton functions within the nucleus in the radius r.

It is well known (see [28]) that two classes of solutions to the Dirac equation for a neutrino of momentum p in the system of spherical coordinates

(r, φ, θ) have the spinor form

$$\psi_{jm}^{(\pm)}(r,\theta,\varphi) = \begin{pmatrix} X_1^{\pm} R_{p,l_u}(r) & Y_{l_u}^{m^-}(\theta,\varphi) \\ \mp X_2^{\mp} R_{p,l_u}(r) & Y_{l_u}^{m^+}(\theta,\varphi) \\ -iX_1^{\mp} R_{p,l_d}(r) & Y_{l_d}^{m^-}(\theta,\varphi) \\ -iX_2^{\pm} R_{p,l_d}(r) & Y_{l_d}^{m^+}(\theta,\varphi) \end{pmatrix}, \tag{8}$$

where $m^+ = m$, $m^- = m - 1$, $l_u = j \mp 1/2$, $l_d = j \pm 1/2$,

$$X_1^{\pm} = \sqrt{\frac{j + 1/2 \pm m^-}{2j + 1 \mp 1}}, \quad X_2^{\pm} = \sqrt{\frac{j + 1/2 \pm m^+}{2j + 1 \pm 1}},$$

j is the quantum number of the total angular momentum (it is half-integer), m is the magnetic quantum number (which is integral), two signs refer to two spin states corresponding to spin directions along (upper sign) and against (lower sign) the total angular momentum, and $R_{p,l}$ is expressed in terms of Bessel functions of a half-integer order (l is an integer) as

$$R_{p,l}(r) = \sqrt{\frac{\pi p}{r}} J_{l+\frac{1}{2}}(pr) \to \frac{\sqrt{2} p^{l+1}}{(2l+1)!!} r^l \tag{9}$$

for $r \to 0$.

In terms of cylindrical coordinates (ρ, φ, z), solutions to the Dirac equation for an electron of energy E in a constant external magnetic field of strength H and a central electric field of the nucleus being considered can be represented in the form

$$\psi_{ns}(t,\rho,\varphi,z) = \sqrt{\frac{eH}{8\pi}} \exp(-iEt) \tag{10}$$

$$\times \begin{pmatrix} \sqrt{1 \pm E_0^{-1}} & J_{n,s,1}^{-}(\rho,\varphi,z) \\ \pm\sqrt{1 \mp E_0^{-1}} & J_{n,s,2}^{+}(\rho,\varphi,z) \\ i\sqrt{1 \pm E_0^{-1}} & J_{n,s,2}^{-}(\rho,\varphi,z) \\ \pm i\sqrt{1 \mp E_0^{-1}} & J_{n,s,1}^{+}(\rho,\varphi,z) \end{pmatrix},$$

where t is time;

$$J_{n,s,h}^{+}(\rho,\varphi,z) \equiv I_{n,s}(\tfrac{1}{2}eH\rho^2) e^{i(n-s)\varphi} \zeta_h(z),$$

$$J_{n,s,h}^{-}(\rho,\varphi,z) = J_{n-1,s,h}^{+}(\rho,\varphi,z), \; h = 1, \; 2;$$

$I_{n,s}$ are Laguerre functions; $\zeta_{1,2}(z)$ are longitudinal-motion functions, which are expressed in terms of Whittaker's functions (the functions ζ_1 and ζ_2 always have opposite parities); n is the number of the respective Landau level; s is the radial quantum number; and $E_0 = \sqrt{1 + 2neH}$. In (10), the transverse (radial) functions in the first and third spinor components are coincident, and so are their counterparts in the second and fourth spinor components. The longitudinal dependences have a different character: it was indicated in [18] that the longitudinal-motion functions in the first and fourth components coincide and that the same is true for their counterparts in the second and third components. For the first Landau level ($n = 0$), the first and third components of the spinor in (10) vanish. This simplifies the respective set of equations, which was solved in [17].

Let us go over from cylindrical to spherical coordinates. At small values of the radius, $eH\rho^2 \ll 1$ (this approximation is legitimate since the nuclear size is much smaller than the Larmor radius), the first term of the expansion of the radial part in a power series in r has the form

$$J_{n,s,h}^{\pm}(r,\theta,\varphi) = \frac{1}{l_e^{\pm}!} \sqrt{\frac{\tilde{n}^{\pm}!}{s!}} \tag{11}$$

$$\times \left(\sqrt{\frac{1}{2} eHr} \sin\theta \right)^{|l_e^{\pm}|} \zeta_h(r\cos\theta) e^{il_e^{\pm}\varphi}$$

$$\approx N_{n,s}^{\pm} \left(\sqrt{\tilde{n}^{\pm} eH} r \sin\theta \right)^{|l_e^{\pm}|} \zeta_h(r\cos\theta) e^{il_e^{\pm}\varphi},$$

where

$$\tilde{n}^{\pm} = n - \tfrac{1}{2} \pm \tfrac{1}{2}, \quad l_e^{\pm} = \tilde{n}^{\pm} - s,$$

$$N_{n,s}^{+} = \frac{1}{(n-s)!} \sqrt{\frac{n!}{s!\,(2n)^{(n-s)}}},$$

$$N_{n,s}^{-} = N_{n-1,s}^{+}.$$

In the semiclassical case, we find at small l_e ($n \gg l_e$) that the coefficients $N_{n,s}$ are independent of the parameters n and H (in order to simplify the presentation, we will henceforth suppress the indices "±," unless this leads to confusion); that is,

$$N_{n,s} = \frac{1}{l_e!} \sqrt{\frac{(n - l_e + 1)(n - l_e + 2)\ldots n}{(2n)^{l_e}}} \sim \frac{1}{l_e! \sqrt{2^{l_e}}}. \tag{12}$$

The characteristic scale of changes in the longitudinal-motion functions $\zeta_{1,2}(z)$ is about the Bohr radius $(\alpha Z)^{-1}$ [17, 18], which, in the superstrong magnetic field being considered, is much larger than the Larmor radius. It follows that, in (11), only the leading term of the expansion in z can be retained for the functions $\zeta_{1,2}(z)$. The spatial-coordinate dependences of the components of the spinor in (10) are contained in the functions $J_{n,s,h}$

(11), which can be expanded in spherical harmonics (4). For the even function $\zeta_h(z)$, we have

$$J_{n,s,h}(r,\theta,\varphi) \tag{13}$$
$$\approx \frac{1}{\sqrt{\pi}} A N_{n,s} \left(\sqrt{neH}\, r\right)^{l_e} \sum_{l'=l_e}^{\infty} a_{l_e l'} Y_{l'}^{l_e}(\theta,\varphi),$$
$$a_{l_1 l_2} = \frac{1}{2}\sqrt{\frac{(l_2-l_1)!\,(2l_2+1)}{(l_2+l_1)!}}$$
$$\times \int_0^{\pi} \sin^{l_1+1}\theta P_{l_2}^{l_1}(\cos\theta)\, d\theta,$$

where $A \equiv \zeta_h(0)$. The amplitude A is determined by the normalization condition. For bound (in the longitudinal direction) states characterized by the longitudinal-motion quantum number κ (where κ is not an integer), this condition has the form [13, 17, 18]

$$A \sim \sqrt{\frac{\alpha Z}{\kappa}}. \tag{14}$$

For excited levels, κ tends to integral values; for the ground state of longitudinal motion (minimal κ), the amplitude and energy depend logarithmically on H [13–18], since κ_0 is a solution to the equation

$$\kappa_0^{-1} = 2\ln\left(\frac{\kappa_0\sqrt{eH}}{2E_0\alpha Z}\right). \tag{15}$$

To a logarithmic accuracy, κ_0 can be disregarded under the logarithm sign. Among the first ten coefficients $a_{ll'}$ ($l, l' \leqslant 3$), the following are nonzero:

$$a_{00} = 1, \quad a_{11} = -\sqrt{\frac{2}{3}}, \tag{16}$$
$$a_{22} = 2\sqrt{\frac{2}{15}}, \quad a_{33} = \frac{-4}{\sqrt{35}}.$$

The contribution of odd states of longitudinal motion to the decay matrix elements is small, since, in the expansion of $J_{n,s,h}$ in spherical harmonics, the ratio of the odd-to-even-state factors is equal to the ratio of the radius of the nucleus to the electron Larmor radius.

For a transition forbidden in order k, we will investigate the first nonzero terms in the expansion of the spatial part of the lepton products in the integrands on the right-hand sides of (3) in spherical harmonics. For individual products, we find from (8) and (10) that

$$\psi_e^{(t)*}\psi_\nu^{(i)} = K^{(ti)}\sqrt{eH} p^{l_\nu+1}\left(\sqrt{neH}\right)^{l_e} \tag{17}$$
$$\times r^{l_e+l_\nu} Y_{l_\nu}^{m}(\theta,\varphi) \sum_{l'=l_e}^{\infty} a_{l_e l'} Y_{l'}^{l_e}(\theta,\varphi),$$

where l_ν is l_u or l_d for, respectively, the "upper" or "lower" components,

$$K^{(ti)} = \frac{A}{2\pi}\sqrt{1 \pm E_0^{-1}}\frac{N^{(t)}X^{(i)}}{(2l_\nu+1)!!}, \tag{18}$$
$$N^{(1)} = N^{(3)} = N^-_{n,\left(n-1-l_e^-\right)},$$
$$N^{(2)} = N^{(4)} = \pm N^+_{n,\left(n-l_e^+\right)},$$
$$X^{(1)} = X_1^{\pm}, \quad X^{(2)} = \mp X_2^{\mp},$$
$$X^{(3)} = X_1^{\mp}, \quad X^{(4)} = X_2^{\pm}.$$

In order to calculate, the matrix elements in question, it is necessary to know the expansion of the nuclear part of the matrix elements in spherical harmonics. For a specific decay, this expansion can be obtained on the basis of the shell model [25, 29]. For each product of the form in (17), the leading term is that which corresponds to the minimum value of degrees of the radius r. In turn, this value is always equal to the sum of the subscript on the neutrino spherical harmonic and the superscript on the electron spherical harmonic. Employing the multiplication rule for spherical harmonics in (5) and taking into account the obvious inequality

$$l_\nu + l_e \geqslant \max\left(|l_\nu - l_e|, m + l_e\right)$$

(which is valid since $l_\nu \geqslant m$ and $l_\nu + l_e \geqslant |l_\nu - l_e|$), we find that the leading term in the expansion of the lepton products has the form

$$\psi_e^{(t)*}\psi_\nu^{(i)} = a_{l_e l_e}\tilde{C}^{l_e+l_\nu}_{l_e,l_e,l_\nu,l_\nu} K^{(ti)}\sqrt{eH} \tag{19}$$
$$\times p^{l_\nu+1}\left(\sqrt{neH}\right)^{l_e} r^{l_e+l_\nu} Y^{l_e+m}_{l_e+l_\nu}(\theta,\varphi).$$

It follows that, for beta decay forbidden in order k, the selection rules for lepton states in a superstrong magnetic field are $l_\nu + l_e^+ = k$ or $l_\nu + l_e^+ = k + 1$, these conditions being formally analogous to the corresponding rules in the absence of external fields. However, a substantial distinction is that, for the electron distribution, l_e is the quantum number of the angular-momentum projection onto the magnetic-field direction rather than the quantum number of the orbital angular momentum (which is not conserved under the geometric conditions being considered).

Using relations (19), we can obtain expressions for all matrix elements of forbidden decays in a superstrong magnetic field. In general, however, this will not lead to a specific result without precise knowledge of nuclear functions: the coefficients in (19) are different for different spinor components (t and i); therefore, the dependences of the different matrix elements in (3) on the magnetic-field strength will also be different. Since the different matrix elements in (3) can make commensurate contributions to the probability of a forbidden beta-decay process, the dependence of the total decay probability on the magnetic-field strength is determined by the ratio of the moments of the nuclear parts of the matrix elements. For these moments, which are individual for each individual decay [25, 29], we have

$$M_{Nk} = \int \Re(\mathbf{r})\, r^k Y_k(\theta, \varphi)\, d^3 r, \qquad (20)$$

where $\Re$ are the corresponding functions $\Re_V$ or $\Re_{Aj}$ (no summation over k is performed here).

3. UNIQUE FORBIDDEN TRANSITIONS

For unique transitions forbidden in order $k > 0$, the dominant contribution to the beta-decay probability comes from only one matrix element—the tensor one in (3), for which the change in the angular momentum is $\Delta I = k + 1$ [26]. In this case (in just the same way as in the case of allowed decays), the nuclear part of the matrix element can be factored outside the sign of summation over lepton states in (1) in calculating the total probability. We then have

$$\lambda = \frac{g^2}{2\pi^3} |M_{Nk}|^2 f_k(Z, Q),$$

where M_{Nk} (20) is the first nonzero moment of the nuclear part of the corresponding matrix element in (3) [25], f_k is the integral Fermi function [26], and Q is the decay endpoint energy. In the unperturbed case (that is, in the absence of a magnetic field), we have

$$f_k(Z, Q) \qquad (21)$$
$$= \int_1^Q F(Z, E) E \sqrt{E^2 - 1}\, (Q - E)^2 S_k(E, Q)\, dE,$$

where F is the Fermi function, which takes into account the distinction between the electron density at the nucleus and the density of free electrons, and S_k is the unperturbed form factor for the unique spectrum of forbiddenness order k [25, 26]. For example, $S_1 = p^2 L_0 + 9L_1$, where the values of $L_{0,1}$ were tabulated in [26]. If one disregards the electric field of the nucleus, then $S_1 \approx p^2 + (E^2 - 1)$. The ratio of the integral Fermi function for unique decay to the Fermi function for allowed decay having the same endpoint energy, f_k/f_0, was also tabulated in [26].

Upon the application of a superstrong magnetic field, it can be found from (19) with allowance for (12) and (18) that the quantity obtained by summing, over all values of l_e, the probability (1) of decay to a specific electron state characterized by the transverse-motion quantum number n and the longitudinal-motion quantum number κ is given by

$$\lambda^H_{n\kappa} = GA^2 eHp^2 \sum_{l=0}^{k} T^k_l (neH)^l p^{2(k-l)}, \qquad (22)$$

$$G = \frac{g^2}{2\pi^3} |M_N|^2,$$

$$T^k_l = \frac{1}{2^l} \left(\frac{a_{ll} \tilde{C}^k_{l,l,k-l,k-l}}{l!\,(2k - 2l + 1)!!} \right)^2.$$

Considering that the sum of the neutrino and electron energies is equal to the beta-decay endpoint energy Q, we obtain

$$\lambda^H_{n\kappa} = GA^2 eH (Q - E(n, \kappa))^2 S^H_k (E(n, \kappa), Q), \qquad (23)$$

where S^H_k is the form factor for forbidden decay in a magnetic field,

$$S^H_k(E, Q) = \sum_{l=0}^{k} T^k_l (neH)^l (Q - E)^{2(k-l)}. \qquad (24)$$

For forbidden decays ($k = 0$, $S_0 = 1$), expression (23) coincides with that which was obtained previously in [18]. In the magnetic field being considered, electrons can occupy states of the continuous spectrum,

$$E_c(n, \kappa) = \sqrt{1 + 2neH + \kappa^2}, \qquad (25)$$

or bound states of the discrete spectrum in the electric field of a nucleus,

$$E_b(n, \kappa) = \sqrt{\frac{1 + 2neH}{1 + (\alpha Z/\kappa)^2}}. \qquad (26)$$

Within the applicability range $1 > eH \gg (\alpha Z)^2$, the following relation holds in (26) for the ground-state level of longitudinal motion [κ_0 in Eq. (15)]:

$$\frac{\alpha Z}{\kappa_0} \sim \sqrt{eH} \left(\frac{2\alpha Z}{\sqrt{eH}} \right) \ln \left(\frac{\sqrt{eH}}{2\alpha Z} \right) < \frac{\ln x}{x} < \frac{1}{2}.$$

Here, x stands for the argument of the logarithm involved.

In the semiclassical case, we find from (26) for $neH \ll 1$ and $\varepsilon_\kappa < \varepsilon_0 \ll 1$, where ε_κ is the binding energy in the state κ and ε_0 is the ground-state binding energy, a spectrum in the form coincident with its nonrelativistic counterpart [15]:

$$E_b(n, \kappa) = 1 + neH - \varepsilon_\kappa.$$

However, expression (26) is also applicable for higher lying Landau levels, $neH > 1$. The binding energy in the ground (κ_0) and excited states is

$$\varepsilon_\kappa = \frac{1}{2} \left(\frac{\alpha Z}{\kappa} \right)^2. \qquad (27)$$

For decays to the continuous spectrum of electrons having a fixed energy E, we find from (23) that

$$\lambda^H_{cE} = GA^2 \sum_{n=1}^{N_{\max}} eH\left(Q-E\right)^2 S^H_k\left(E,Q\right), \quad (28)$$

where

$$N_{c\,\max} = \left(E^2-1\right)/2eH \quad (29)$$

and κ is determined for each value of n from the condition

$$\kappa_n = \pm\sqrt{E^2-1-2neH}. \quad (30)$$

In the semiclassical case, $n \gg 1$, we go over from the sum in (28) to an integral with respect to $x = neH$. This yields

$$\lambda^H_{cE} = GA^2\left(Q-E\right)^2 \sum_{l=0}^{k} T^k_l \quad (31)$$
$$\times \int\limits_0^{\left(E^2-1\right)/2} x^l\left(Q-E\right)^{2(k-l)}dx$$
$$= GA^2 \sum_{l=0}^{k} \frac{T^k_l}{(l+1)\,2^{l+1}}$$
$$\times \left(Q-E\right)^{2(k-l)+2}\left(E^2-1\right)^{l+1}.$$

We can see that the probability of decay to the continuous spectrum of electrons having a fixed energy E is independent of the magnetic-field strength and so is therefore the total probability of decay to the continuous spectrum. In just the same way as in the case of allowed decays [18], the reason for this is that, although the density of states at a specific energy increases in direct proportion to the field strength H [see Eq. (10)], the number of possible states decreases in inverse proportion to H [see Eq. (29)].

4. DECAY TO A BOUND STATE

Let us consider beta decay to bound states formed in an external magnetic field and the Coulomb field of a nucleus. A superstrong magnetic field changes qualitatively the structure of electron bound states in the nuclear field. For each level of electron transverse motion (Landau level), there arises the spectrum of bound states (26), to which beta decay may occur. These states were absent in the unperturbed case (without a magnetic field). The quantity obtained by summing, over all Landau levels, the probability of decay to a specific state of longitudinal motion is given by

$$\lambda^H_{b\kappa} = G\frac{\alpha Z}{\kappa} \sum_{n=1}^{N_{0\,\max}} eH\left(Q-E_b\left(n,\kappa\right)\right)^2 \quad (32)$$
$$\times S^H_k\left(E_b\left(n,\kappa\right),Q\right),$$
$$N_{b\,\max} = \frac{Q^2\left(1+2\varepsilon_\kappa\right)-1}{2eH}.$$

For a fixed level of transverse motion, the sum over all possible Landau levels in (32) is weakly dependent on the magnetic-field strength, which appears in this sum only indirectly, through the dependence $\varepsilon_0(H)$.

Beta decay may occur not only to the ground state but also to excited states of longitudinal motion. It is noteworthy that, for a unionized atom in a magnetic field, the atomic electrons may occupy only the lowest levels of bound states, the total number of levels of the ground bound state being quite large, $N_{b\,\max} \gg 1$. The probability of decay to bound states [see Eq. (32)] becomes higher with increasing magnetic field for two reasons: first, the amplitude A (14) grows; second, the decay endpoint energy effectively increases $\left(Q \to \tilde{Q}\right)$—from a comparison of Eqs. (29) and (32), we obtain

$$\tilde{Q} = Q\sqrt{1+2\varepsilon_\kappa}. \quad (33)$$

In relation to allowed decays, the probability of forbidden decays increases additionally because of the growth of the form factor S^H_k. At low decay energies, $q = Q-1 \ll 1$, we find from (33) that

$$\tilde{q} \equiv \tilde{Q} - 1 \approx q + \varepsilon_\kappa. \quad (34)$$

The form factor for decay to the ground bound state is given by

$$S^H_k\left(E_b\left(n,\kappa_0\right),Q\right) \approx \sum_{l=0}^{k} T^k_l\left(neH\right)^l \quad (35)$$
$$\times \left(Q-1-neH+\varepsilon_0\right)^{2(k-l)}$$
$$= S^H_k\left(E_c\left(n,0\right),Q+\varepsilon_0\right).$$

At low decay endpoint energies, we go over from the sum in (32) to an integral with respect to $x = neH$ and ultimately obtain

$$\lambda^H_{b\kappa} = G\frac{\alpha Z}{\kappa} \sum_{l=0}^{k} T^k_l\, l!\frac{(2k-2l+2)!}{(2k-l+3)!}\tilde{q}^{2k-l+3}. \quad (36)$$

Since the matrix elements that determine the coefficient G in (22) are independent of the magnetic field, the relative increase in the probability $\left(\lambda^H_b/\lambda\right)_k$ for unique beta decay forbidden in order k is independent of the nuclear matrix elements. In terms of the relative increase $\left(\lambda^H_b/\lambda\right)_0$ in the probability of allowed beta decay, we have

$$\eta_k \equiv \left(\frac{\lambda^H_b}{\lambda}\right)_k \bigg/ \left(\frac{\lambda^H_b}{\lambda}\right)_0 \quad (37)$$

$$= \frac{f_0}{f_k} \sum_{l=0}^{k} T_l^k \cdot 3l! \frac{(2k-2l+2)!}{(2k-l+3)!} \tilde{q}^{2k-l}.$$

If the decay endpoint energy is much lower than the energy of the ground bound state of longitudinal motion, $q \ll \varepsilon_0 \ll 1$, then the following qualitative estimate can be obtained from (37) with the aid of the approximate relation $f_k/f_0 \propto q^k$ [26]:

$$\eta_k \propto \left(\frac{\varepsilon_0}{q}\right)^k \propto \frac{(\alpha Z)^{2k}}{q^k} \ln^{2k}\left(\frac{\sqrt{eH}}{2\alpha Z}\right). \qquad (38)$$

Thus, one can see that, at low endpoint energies, the form factor for unique forbidden beta decay increases as Z increases and as q decreases; also, this form factor grows logarithmically with increasing magnetic-field strength.

5. CONCLUSIONS

The application of a superstrong external magnetic field to an atom leads to an increase in the probability of forbidden electron beta decays of nuclei owing to decay to electron bound states. This increase is more pronounced than in the case of allowed decays [18], since not only does the density of vacant electron states at the nucleus involved become higher, but the decay form factor also increases. By way of example, we can compare the main channels of the decays of ^{134}Cs ($4^+ \to 4^+$ allowed transition, 658 keV, $T_{1/2} = 2$ yr) and ^{137}Cs ($7/2^+ \to 11/2^-$ unique transition forbidden in the first order, 514 keV, $T_{1/2} = 30$ yr), where the decay endpoint energies are close. A numerical analysis of formulas (32) and (35) ultimately reveals that, within the range of applicability of the model being considered, the ratio of the probabilities for the decay of ^{137}Cs and ^{134}Cs must increase by a factor of 3.

At low endpoint energies, $q < eH$, beta decay may occur only to the first Landau level of transverse motion. In this case, the probability of allowed beta decays to a bound state increases in direct proportion to the nuclear charge Z and in direct proportion to the magnetic-field strength [10], while, in the absence of a magnetic field, the probability of decay to a bound state is proportional to Z^3 [11]. The probability of allowed decays ceases to be dependent on the magnetic field if the decay endpoint energy falls within the range $eH \ll q \ll 1$.

The dependence of the form factor for forbidden decays on the nuclear charge and the decay energy [see Eq. (38)] manifests itself if the decay energy is much lower than the binding energy for the ground-state level of longitudinal motion, $q \ll \varepsilon_0$ (ε_0 can in general be about eH). In this case, the form factor for unique forbidden decay increases as the nuclear charge increases or as the decay energy decreases; also, this form factor grows weakly with increasing magnetic-field strength.

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