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NUCLEI
Theory

Increase in the Probability of Allowed Electron Beta Decays in a Superstrong Magnetic Field

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Abstract—The effect of a superstrong constant uniform magnetic field, $H \gg H_0 = cm_e^2e^3/\hbar^3$, on the probability of allowed electron beta decays is considered. It is shown that, for an atom whose nucleus is β^--active and which is placed in a superstrong magnetic field, the β^--decay probability increases owing to the enhancement of β^- decay to a bound state of the electron. The effect is operative both for the nucleus of a fully ionized atom and for the nucleus of a neutral atom.

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INTRODUCTION

Changes in atomic electron states upon placing an atom in a superstrong constant uniform external magnetic field were examined in [1–5]. From the results obtained in [1–5], it follows that this leads to an increase in the density of electron states and to a change in the ionization energy of the atom. In the present study, it will be shown that, for an atom featuring a β^--active nucleus, this rearrangement of atomic electron states increases the β^--decay probability.

The effect of the electromagnetic-wave field on the probability of the β^- decay of a nucleus was studied in [6–8]. A constant magnetic field was considered there as a particular case. The conclusion drawn in [6, 7] is that, in an external magnetic field, the total probability of the β^- decay of a nucleus undergoes no changes, with the exception of the emergence of a small quantum correction in the case of decay to the lowest Landau level, this correction leading to a decrease in the β^--decay probability.

It should be emphasized that, in [6–8], the electric field of the nucleus being considered was disregarded—that is, the consideration there was in fact performed for the decay of the nucleus of a fully ionized atom without allowance for β^- decay to bound electron states. However, the β^- decay of the nucleus of a fully ionized atom differs from the β^- decay of a neutral atom owing primarily to decay to a bound state—that is, a decay process where a β^- electron is produced in an unfilled atomic orbit rather than being emitted from the atom (see [9, 10] for the theory of decay to a bound state, [11, 12] for the theory of the decay of a fully ionized atom, [13] for experimental data concerning the observation of β^- decay to a bound state of ^{187}Re, and [14] for an overview of studies devoted to tritium β^- decay). In the case where the initial nucleus undergoing β^- decay is an emitter of delayed neutrons, decay to a bound state leads to a change in the delayed-neutron fraction [15].

Thus, it is presently well known that the density of electron states in the vicinity of a nucleus (and, hence, the probability of decay to a bound state) depends on an external electromagnetic field, but this circumstance was disregarded in [6–8]. Moreover, the ionization energy of a neutral atom (or of a not fully ionized atom) changes upon placing this atom in a superstrong magnetic field [4, 5], and this leads to a change in the β^--decay endpoint energy [16, 17] and, hence, to a change in the β^--decay probability.

1. ELECTRON BETA DECAY TO A BOUND STATE

The β^--decay probability λ is the sum of the probabilities of decay to a continuous electron spectrum, λ_c, and decay to a bound state, λ_b [9, 10]. The decay to a bound state is especially important for decays of nuclei of fully ionized atoms [11, 12]. For the decay of a neutral unperturbed atom, the probability λ_b is small, since all lower electron orbits are occupied, while upper ones are characterized by a very low electron density in the vicinity of nuclei. However, it will be shown in the present study that, upon placing

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an atom in a superstrong magnetic field, the density of excited atomic electron states in the vicinity of its nucleus increases, with the result that the probability of β^- decay to a bound state becomes sizable not only for the β^- decay of the nucleus of a fully ionized atom but also for the decay of the nucleus of a neutral atom.

Unless otherwise stated, we will use below the system of relativistic units where $\hbar = c = m_e = 1$, with $\hbar$, c, and m_e being, respectively, the Planck constant, the speed of light in a vacuum, and the electron mass. The probability of electron beta decay via allowed transitions to a continuous spectrum is proportional to the integrated Fermi function, which characterizes the phase space of final states [18]. Specifically, we have

$$\lambda_c = \frac{g^2|M|^2}{2\pi^3} f(Z,U), \qquad (1)$$

$$f(Z,U) = \int_1^U F(Z,E)E\sqrt{E^2-1}(U-E)^2 dE,$$

where g is the weak coupling constant; M is the nuclear matrix element; Z is the charge number of the nucleus involved; U is the β^--decay endpoint energy; and F is the ratio of the density of final electron states in the vicinity of the nucleus with allowance for external and atomic electromagnetic fields to the density of states for free particles,

$$F(Z,E) = \frac{2\pi^2}{p_e^2} \sum_i \psi_i^+ \psi_i. \qquad (2)$$

Here, p_e is the momentum of the electron whose total energy is E, ψ_i is a spinor that describes an electron state characterized by the quantum-number set i, and the plus symbol in the superscript (ψ_i^+) denotes Hermitian conjugation. The sum in (5) is taken over all possible states of total energy E.

For allowed β^- decays, λ_c and λ_b are proportional to the same nuclear matrix element [9, 10]:

$$\lambda_b(E_j) = \frac{g^3|M|^2}{\pi} \sum_i \psi_i^+ \psi_i (U - E_j)^2. \qquad (3)$$

In just the same way as in (2), the sum here is taken over all bound states of total electron energy E_j. The total β^--decay probability is

$$\lambda = \lambda_c + \sum_j \lambda_b(E_j). \qquad (4)$$

In order to calculate the probability of β^- decay to bound states, it is necessary to know the electron distribution in the central electric field of the nucleus and a constant uniform magnetic field. In general, the variables of the problem at hand cannot be separated. We are interested in the case of superstrong magnetic fields,

$$H \gg H_0 = \frac{cm_e^2 e^3}{\hbar^3} \approx 2.35 \times 10^9 \text{ Oe}, \qquad (5)$$

that is, fields in which the Larmor radius of an electron, r_L, is much smaller than the Bohr radius R_B,

$$r_{\rm L} = \sqrt{\frac{\hbar c}{eH}} = \sqrt{\frac{cm_e^2 e^3}{\hbar^3 H}} \frac{\hbar^2}{m_e e^2} \qquad (6)$$
$$= \sqrt{\frac{H_0}{H}} \frac{\hbar^2}{m_e e^2} \ll \frac{\hbar^2}{m_e e^2} = R_{\rm B},$$

and in which the cyclotron-rotation energy $\frac{1}{2}\hbar\Omega_C$ is much greater than the ionization potential of the hydrogen atom,

$$\frac{1}{2}\hbar\Omega_C = \hbar\frac{eH}{2m_e c} \gg \frac{m_e e^4}{2\hbar^2} = I_{\rm H}. \qquad (7)$$

In this case, the electric field of the nucleus is a small correction to the external magnetic field.

2. ELECTRON IN A CENTRAL ELECTRIC AND A SUPERSTRONG UNIFORM MAGNETIC FIELD

In the nonrelativistic approximation, the problem being considered was solved in [1, 2, 5]. Those studies were devoted to examining the effect of superstrong magnetic fields on the deformation of atomic electron shells. The motion of an electron in a superstrong constant uniform magnetic field and a central electric field is a superposition of the following two motions: (i) motion in the plane orthogonal to the magnetic field (motion over Landau levels) and (ii) motion along the magnetic-field direction (one-dimensional Coulomb motion).

It is noteworthy that bound states of an electron (along the magnetic-field direction) exist for all Landau levels, including rather high ones (that is, those at high energies). This means that, in the case of superstrong magnetic fields, it is necessary to employ the relativistic approximation both for a continuous spectrum and for bound states. In the relativistic approximation, the problem of a hydrogen-like orbit in a superstrong magnetic field was solved by Krainov [19], who considered the case of stronger magnetic fields in which the Larmor radius is much smaller than the electron Compton wavelength.

In the approximation of a superstrong magnetic field [see Eq. (5)], we will consider the electric field as a perturbation that affects the motion of an electron in the magnetic field over the Landau levels. We will use the well-known solution to the Dirac equation in a constant uniform magnetic field [20]. We will write the Dirac equation in the system of cylindrical coordinates (r, ϕ, z) for an electron in the electromagnetic field that is a superposition of the central electrostatic field of a nucleus characterized by a charge number Z and positioned at the origin of coordinates and a constant magnetic field of strength H along the z axis. We have

$$\left\{ i\partial_t - \boldsymbol{\alpha}_0 + \frac{\alpha Z}{\sqrt{r^2+z^2}} + i\boldsymbol{\alpha}_3\partial_z + i\boldsymbol{\alpha}_1 e^{\pm i\varphi}\left[\partial_r \pm \frac{i}{r}\partial_\varphi \mp \gamma r\right]\right\}\psi = 0, \quad (8)$$

where $\gamma = eH/(2c\hbar)$ [the condition in (6) means that $\gamma \gg \alpha^2$], α is the fine-structure constant, $\partial_\mu \equiv \partial/\partial\mu$ is a partial derivative with respect to the corresponding coordinate, the upper (lower) sign in (8) refers to the action on the first and third (second and fourth) components of the spinor ψ, and $\boldsymbol{\alpha}_k$ stands for the Dirac α matrices.

We seek solutions to Eq. (8) in the form [20]

$$\psi_{ns} = \frac{1}{\sqrt{2\pi}}\exp[-iEt + i(n-s)\varphi]\sqrt{2\gamma} \times \begin{pmatrix} \chi_{1,ns}(z)I_{n-1,s}(\gamma r^2)e^{-i\varphi} \\ i\chi_{2,ns}(z)I_{n,s}(\gamma r^2) \\ \chi_{3,ns}(z)I_{n-1,s}(\gamma r^2)e^{-i\varphi} \\ i\chi_{4,ns}(z)I_{n,s}(\gamma r^2) \end{pmatrix} \quad (9)$$

and require fulfillment of the normalization condition

$$\sum_{\mu=1}^{4}\int_{-\infty}^{\infty}\chi_\mu^+(z)\chi_\mu(z)dz = 1$$

for the discrete spectrum of the longitudinal motion or fulfillment of the normalization condition

$$\sum_{\mu=1}^{4}\int_{-\infty}^{\infty}\chi_\mu^+(z,k_1)\chi_\mu(z,k_2)dz = \delta(k_1-k_2)$$

for the continuous spectrum. Here, I are radial functions expressed in terms of Laguerre polynomials Q as

$$I_{n,s}(\rho) = \frac{1}{\sqrt{n!s!}}e^{-\rho/2}\rho^{(n-s)/2}Q_s^{n-s}(\rho), \quad (10)$$

where

$$Q_k^m(\rho) = e^\rho\rho^{-m}\frac{d^k}{d\rho^k}(e^{-\rho}\rho^{k+m}),$$

n is the principal quantum number, s is the radial quantum number, $I_{n,n}(0) = 1$ and $I_{n,s}(0) = 0$ for $n \neq s$, and $I_{-1,0}(\rho) \equiv 0$. In order to calculate the change in the probability of the allowed β^- decay, as given by Eqs. (2) and (3), it is necessary to estimate the change in the electron density at the position of the nucleus—that is, at the origin of coordinates. We will make use of the condition that the motion of an electron along the magnetic-field direction is slow (adiabatic) in relation to its rotation in the plane orthogonal to the magnetic field, this being equivalent to the smallness of the longitudinal-motion energy in relation to the transverse-motion energy (7).

For a zero-order approximation, we consider a solution to Eq. (8) without an electric field. In this case, the functions $\chi_\mu(z)$ in (9) assume the form [20]

$$\begin{pmatrix}\chi_1\\ \chi_2\\ \chi_3\\ \chi_4\end{pmatrix} = \frac{e^{ik_z z}}{4\sqrt{\pi}}\begin{pmatrix}\sqrt{1+\sigma/E_0}(A_1+A_2)\\ \sigma\sqrt{1-\sigma/E_0}(A_2-A_1)\\ \sqrt{1+\sigma/E_0}(A_1-A_2)\\ \sigma\sqrt{1-\sigma/E_0}(A_1+A_2)\end{pmatrix}, \quad (11)$$

where $\sigma = \pm 1$ is a number that characterizes the electron spin state (spin projection onto the magnetic-field direction) and

$$A_1 = \sqrt{1+k_z/E},\ A_2 = \sigma\sqrt{1-k_z/E} \quad (12)$$

with

$$E = \sqrt{1+k_z^2+4n\gamma},\ E_0 = \sqrt{1+4n\gamma}.$$

For a solution unperturbed by the electric field, we define the transverse-motion-averaged Coulomb potential as

$$\Phi_{ns}(z) = \int_0^{2\pi}\int_0^{\infty}\left(\psi_{ns}^+\frac{\alpha Z}{\sqrt{z^2+r^2}}\psi_{ns}\right)d\varphi r dr. \quad (13)$$

Substituting the solution specified by Eqs. (9) and (11) into (13) and going over to the variable $\rho = \gamma r^2$, we arrive at

$$\Phi_{ns}(z) = \frac{1}{2}\alpha Z\sqrt{\gamma} \times \int_0^\infty \frac{\left(1+\frac{\sigma}{E_0}\right)I_{n-1,s}^2(\rho) + \left(1-\frac{\sigma}{E_0}\right)I_{n,s}^2(\rho)}{\sqrt{\gamma z^2+\rho}}d\rho. \quad (14)$$

On the basis of (14), one can readily establish the following properties of the functions $\Phi_{ns}(z)$:

(i) The functions $\Phi_{ns}(z)$ are even.

(ii) In the region $z > 0$, they are monotonic [the derivatives $\Phi'_{ns}(z)$ are negative].

(iii) At the origin, $z = 0$, the functions $\Phi_{ns}(z)$ are finite, since the integral $\int_0^\infty \rho^{-1/2}I_{n,s}^2(\rho)d\rho$ is finite, and $\Phi_{ns}(0) = C_{ns}\alpha Z\sqrt{\gamma}$, the constants C_{ns} being henceforth independent of the parameters Z and γ.

(iv) The asymptotic behavior of $\Phi_{ns}(z)$ for $z \gg r_{\rm L}$ is independent of the quantum numbers n and s: $\Phi_{ns} \to \alpha Z/|z|$.

In just the same way as in the nonrelativistic case (see [5]), we can therefore derive the required estimate by using the approximation

$$\Phi_{ns}(z) \approx \frac{\alpha Z}{|z| + a_{ns}}, \qquad (15)$$

where the parameter $a_{ns} = \alpha Z/\Phi(0) = (C_{ns}\sqrt{\gamma})^{-1}$ is on the order of the Larmor radius of the electron [see Eq. (6)].

By using this approximation, we find from (8) and (9) that the functions χ_μ satisfy the equations

$$[E \mp 1 + \Phi_{ns}(z)]\chi_{1,3} + i\partial_z\chi_{3,1} - \sqrt{4n\gamma}\chi_{4,2} = 0, \qquad (16)$$
$$[E \mp 1 + \Phi_{ns}(z)]\chi_{2,4} - i\partial_z\chi_{4,2} - \sqrt{4n\gamma}\chi_{3,1} = 0.$$

In the field configuration considered here, the polarization tensor is not a conserved quantity, in contrast to what occurs in the case of a constant magnetic field [20] in the absence of an electric field. Nevertheless, the projection of the total angular momentum onto the magnetic-field direction, $J_z = -i\partial_\varphi + \frac{1}{2}\sigma_3$ (σ_3 is a Pauli matrix), remains an integral of motion. The solution specified by Eq. (9) is an eigenfunction of the operator J_z, the corresponding eigenvalue being $(n - s - 1/2)$. In order to construct a general solution, we can make use of a parameter that is similar to the polarization σ (11). In the set of Eqs. (16), we make the substitution

$$\begin{pmatrix}\chi_1\\ \chi_2\\ \chi_3\\ \chi_4\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{1+\sigma/E_0}f_1(z)\\ \sigma\sqrt{1-\sigma/E_0}f_2(z)\\ -\sqrt{1+\sigma/E_0}f_2(z)\\ \sigma\sqrt{1-\sigma/E_0}f_1(z)\end{pmatrix}, \qquad (17)$$

where, as in (11), $\sigma = \pm 1$ (in the present case, this is a parameter rather than the spin projection) and $E_0 = \sqrt{1 + 4n\gamma}$. Going over from the variable z to the variable $x = k(|z| + a_{ns})$ and retaining the leading term in the expansion of $\Phi(z)$ in powers of $1/z$, we find that the functions $f_{1,2}$ satisfy the set of equations

$$\left[\frac{E + \sigma E_0}{k} + \frac{\alpha Z}{x}\right] f_2(x) = i f_1'(x), \qquad (18)$$
$$\left[\frac{E - \sigma E_0}{k} + \frac{\alpha Z}{x}\right] f_1(x) = i f_2'(x).$$

For bound states (discrete spectrum), in which case $E < E_0$, this set of equations reduces to the Whittaker equation with $k = 2E\alpha Z/m$. Specifically, we have

$$f''_{xx}(x) + \left[-\frac{1}{4} + \frac{m}{x}\right] f(x) = 0 \qquad (19)$$

under the condition

$$E^2 = E_0^2\left[1 + \left(\frac{\alpha Z}{m}\right)^2\right]^{-1} = \frac{1 + 4n\gamma}{1 + (\alpha Z/m)^2} < E_0^2. \qquad (20)$$

Solutions to this equation were studied in [1, 5, 19]. In the semiclassical case, the spectrum given by Eq. (20) is broken down into the sum of the transverse-motion energy $2n\gamma$ and the longitudinal-motion energy $-1/2(\alpha Z/m)^2$. For the discrete spectrum, the Whittaker functions $W_{m,1/2}$ are solutions to Eq. (19). The quantum number m characterizing longitudinal motion (it must not be integral) is determined from the requirement that the solution $f(z)$ be continuous at $z = 0$—that is, at $x_0 = 2E\alpha Z a_{ns}/m$, $W_{m,1/2}(x_0) = 0$ for the odd functions $f(z)$ or $W'_{m,1/2}(x_0) = 0$ for their even counterparts. We are interested in the nonzero density of electron states at the position of the nucleus—that is, the even functions $f(z)$, for which m is determined from the condition

$$m = 2E\alpha Z\frac{a_{ns}}{q_m} = \frac{2E\alpha Z}{C_{ns}\sqrt{\gamma}q_m}, \qquad (21)$$

where q_m are zeros of the derivative of the Whittaker function $W_{m,1/2}$.

For the continuous spectrum (the respective motions are not localized), in which case $E > E_0$, the set of Eqs. (18) leads to the equation

$$f''_{xx} + \left[1 + \frac{2E\alpha Z}{kx}\right] f = 0, \qquad (22)$$

the dispersion equation for it having the form

$$E^2 = E_0^2 + k^2 = 1 + 4n\gamma + k^2. \qquad (23)$$

At large distances, the asymptotic behavior of solutions to Eq. (22), $f(x) \equiv f(kz) \sim e^{ikz}$, coincides with the solutions in (11) in the absence of an electric field. Similarly to (21), the condition of continuity of the even functions requires fulfillment of the relation

$$f'(ka_{ns}) \equiv f'(k/C_{ns}\sqrt{\gamma}) = 0. \qquad (24)$$

3. PROBABILITY OF ALLOWED β^- DECAYS IN A MAGNETIC FIELD

The solutions obtained above will now be analyzed with the aim of estimating the change in the β^--decay probability upon the application of a superstrong magnetic field.

3.1. *Absence of an Electric Field*

Only continuum states are possible in this case, which is specified by Eqs. (9), (11), and (12). Precisely two states corresponding to $s = n$ and $s = n - 1$ and having a nonzero density at the position of the nucleus involved ($r = 0$, $z = 0$) exist at each value of the principal quantum number n (this is not so only for the $n = 0$ level, in which case there is one state, that of $s = 0$). The density of each pair of such states at the position of the nucleus is independent of the energy E and is proportional to the magnetic field [see Eq. (9)],

$$\psi_n^+\psi_n = \gamma/(2\pi^2). \tag{25}$$

From the spectrum in (12), it is clear that, at a given total electron energy E, the principal quantum number can take values in the range from 0 to $N_{\max}$, where

$$N_{\max} = (E^2 - 1)/(4\gamma). \tag{26}$$

It follows that, at a rather high energy, $E \gg \gamma$, the density of continuum states in the absence of an electric field is independent of the magnetic-field strength. Specifically,

$$\psi_E^+\psi_E \sim \sum_{n=1}^{N_{\max}} \psi_n^+\psi_n \tag{27}$$
$$\sim N_{\max}\psi_n^+\psi_n \sim (E^2 - 1)/(8\pi^2),$$

which is in accord with the results presented in [6].

3.2. *Continuous Spectrum in the Electric Field of a Nucleus*

Equation (22) does not explicitly involve the magnetic-field parameter γ. Therefore, the change in the magnetic field according to $\gamma \to \gamma\lambda$ leads to a similar change in the solutions; that is, the function $f(x)$ does not change, but the values of the wave vector k, which are determined by Eq. (24), increase as follows: $k \to \tilde{k} = k\sqrt{\lambda}$. Using the normalization condition for the function $\chi(z)$ and considering that the Dirac delta function possesses the property $\delta(k\sqrt{\lambda}) = \delta(k)/\sqrt{\lambda}$, we find that, in response to a change in the magnetic field, the amplitude changes as $\chi_0 \to \tilde{\chi}_0 = \chi_0\sqrt[4]{\lambda}$:

$$\delta(k_1 - k_2) = \int_{-\infty}^{\infty} \tilde{\chi}^+(z, k_1)\tilde{\chi}(z, k_2)dz \tag{28}$$
$$= \frac{\tilde{\chi}_0^2}{\chi_0^2}\int_{-\infty}^{\infty} \chi^+(z, \tilde{k}_1)\chi(z, \tilde{k}_2)dz$$
$$= \frac{\tilde{\chi}_0^2}{\chi_0^2}\delta(\tilde{k}_1 - \tilde{k}_2) = \frac{\tilde{\chi}_0^2}{\sqrt{\lambda}\chi_0^2}\delta(k_1 - k_2).$$

It follows that, instead of (25), we obtain

$$\psi_n^+\psi_n = \frac{\gamma}{2\pi^2}\sqrt{\frac{\gamma}{\gamma_0}}, \tag{29}$$

where $\gamma_0 \sim \alpha^2$, which, with allowance for (26), leads to the magnetic-field dependence of the density of continuum states:

$$\psi_E^+\psi_E \sim \sum_{n=1}^{N_{\max}} \psi_n^+\psi_n \sim \frac{E^2 - 1}{8\pi^2}\sqrt{\frac{\gamma}{\gamma_0}}. \tag{30}$$

In other words, an increase in the magnetic-field strength leads not only to the compression of the electron cloud owing to a decrease in the Larmor radius but also to an increase in the effective electric-field potential (14). This in turn leads to the compression of the electron distribution in the direction along the magnetic field.

3.3. *Discrete Spectrum (Bound States) in the Electric Field of the Nucleus*

The solution to Eq. (19) does not involve the features of the field. From the similarity of the solutions $f(x)$ and the normalization condition for $\chi(z)$,

$$\int_{-\infty}^{\infty} \chi^+(z)\chi(z)dz \tag{31}$$
$$= 2\chi_0^2\int_{x_0}^{\infty} f^+(x,k)f(x,k)\frac{dx}{k} \approx \frac{2\chi_0^2}{k} = 1,$$

we find that the function $\chi(z)$ has the form

$$\chi_0^2 = \frac{E\alpha Z}{m}, \tag{32}$$

where m is the quantum number of longitudinal motion. Therefore, the density of states in the discrete spectrum (characterized by the quantum numbers n and m) at the position of the nucleus is given by

$$\psi_{nm}^+\psi_{nm} = \frac{1}{\pi}\gamma\frac{E\alpha Z}{m} > \frac{1}{\pi}\gamma\frac{\alpha Z}{m}. \tag{33}$$

It is well known that, in the three-dimensional problem of motion in the Coulomb potential (without a magnetic field), which is spherically symmetric, the density of a hydrogen-like orbit at the center is

$$\psi_m^+\psi_m \sim \frac{1}{\pi}\left(\frac{\alpha Z}{m}\right)^3. \tag{34}$$

A comparison of expressions (33) and (34) reveals that, in the absence of a magnetic field, the density of excited-electron orbits at the position of the nucleus decreases very fast with increasing orbit number in proportion to m^{-3} (34); therefore, there is virtually no decay to excited bound states in this case. In the presence of an external magnetic field, β^- decay to bound states becomes significant in the case of transitions to orbits of large quantum number, which are free for a neutral atom inclusive [see Eq. (33)]. Formally, the sum $\sum 1/m$ is divergent, but, in practice, summation should be cut off at levels for which the characteristic longitudinal size of the electron-density variation reaches the characteristic scale of the external-magnetic-field variation.

Substituting (33) into (3), we find that the relative increase in the probability of electron beta decay because of transitions to bound states, λ_{bH}/λ_c, in the presence of an external magnetic field H is

$$\frac{\lambda_{bH}}{\lambda_c} \sim 2\pi(\gamma\alpha Z)\sum_{n=1}^{N_{\max}}\sum_{m=1}^{M}\frac{1}{m}\frac{(U-E_{nm})^2}{f(Z,U)}, \quad (35)$$

where E_{nm} is the energy of the bound state specified by the quantum numbers n and m (20), U is the endpoint energy of electron beta decay, and $N_{\max} = (U-1)/(4\gamma)$.

The relative increase in the probability of electron beta decay because of transitions to bound states in the case of a full ionization of the atom being considered in the absence of a magnetic field is [9, 10]

$$\frac{\lambda_b}{\lambda_c} \sim 2\pi(\alpha Z)^3\frac{(U-1+\varepsilon)^2}{f(Z,U)}, \quad (36)$$

where ε is the electron binding energy in the respective orbit, $\varepsilon \ll 1$. A comparison of (35) and (36) reveals that the probability of decay to a bound state in a superstrong magnetic field, $\gamma \gg \alpha^2$ [see Eq. (6)], exceeds the probability of decay to a bound state in a fully ionized atom for $\gamma > (\alpha Z)^2$.

The above estimates were obtained for a hydrogen-like orbit. This approximation is also applicable to highly excited (Rydberg) states if one electron of a multielectron atom is in an excited state [21]. Since the increase in the probability of electron beta decay is due primarily to transitions to highly excited bound states, the above conclusion is qualitatively applicable to the decay of a nucleus in an atom.

The β^--decay probability may change not only because of the increase in the density of free electron states but also because of the change in the endpoint energy of β^- decay.

4. CHANGE IN THE ENDPOINT ENERGY OF ELECTRON BETA DECAY IN A SUPERSTRONG MAGNETIC FIELD

In the electron beta decay of a nucleus entering into the composition of an atom and occurring in the external magnetic field, the β^--decay endpoint energy U differs from its counterpart U_0 for the respective nucleus in an unperturbed atom [16, 17]. Since a nucleus surrounded by electrons interacting with it appears both in the initial and in the final state, the β^--decay endpoint energy U is the difference of the total internal energies of the initial and final states of the system with allowance for the ionization energy of the atom; that is,

$$U_0 = U_n + [I_f^0 - I_i^0], \quad (37)$$
$$U = U_n + [I_f^H - I_i^H],$$
$$U = U_0 - [I_f^0 - I_i^0] + [I_f^H - I_i^H],$$

where U_n is the difference of the nuclear energies, $I > 0$ is the total ionization energy of the atom being considered, the superscripts label respective quantities for the unperturbed atom ("0") or the atom in a magnetic field ("H"), and the subscripts label the analogous quantities in the final atomic (ion) state appearing as the product of electron beta decay ("f") or in the initial atomic state ("i").

Within the Thomas–Fermi model, the total ionization potential of an unperturbed multielectron atom whose charge number is Z is given by [22]

$$I^0(Z) \approx 20.8Z^{7/3} \text{ [eV]}, \quad (38)$$

whence it follows that

$$I_f^0(Z) - I_i^0(Z) \approx 48.5Z^{4/3} \text{ [eV]}. \quad (39)$$

The behavior of the electron shell of an atom in a superstrong magnetic field [see (5)] was considered in [2, 3]. Under the condition $H \gg H_0Z^3$, the total ionization energy of an atom (or an ion) having a charge number Z and containing K electrons is [2]

$$I^H(Z,K) \approx \frac{K}{8}L^2(4Z-K+1)^2, \quad (40)$$
$$L = \frac{1}{2}\ln\frac{H}{H_0Z^3}.$$

To a logarithmic precision, $L \approx \text{const}$, we find from (40) that

$$I_f^H(Z) - I_i^H(Z) \approx 3L^2Z(Z+1) \quad (41)$$
$$\approx 81.6L^2Z(Z+1) \text{ [eV]}.$$

From a comparison of expressions (39) and (41), one can see that the ionization energy of an atom in a rather strong external magnetic field grows with increasing charge number of the nucleus faster than its counterpart for the respective unperturbed atom. Therefore, the endpoint energy U for the electron beta decay of the nucleus of an atom in a superstrong external magnetic field [see (37)] is higher than the respective β^--decay endpoint energy for the unperturbed atom.

An increase in the endpoint energy of electron beta decay leads to an increase in its probability (1). The effect may be substantial for decays characterized by a low endpoint energy. It is well known from [13] that a complete ionization of the ^{187}Re atom leads to the increase in the endpoint energy of electron beta decay from 2.66 keV for the neutral atom to 72.97 keV. This leads to the opening of the channel of decay to the 9.75-keV excited level of ^{187}Os, with the result that the probability of ^{187}Re electron beta decay increases by a factor of 10^9. From (41), it follows that, in the case where the model specified by Eqs. (40) is applicable, the probability of ^{187}Re electron beta decay will increase to the same extent upon placing the neutral atom in a superstrong magnetic field such that $L \sim 1/2\,(H > 3H_0Z^3 \sim 3\times 10^{15}$ Oe).

In the decay of the nucleus of a completely ionized atom and in the cases where the magnetic field is not very high, $H < H_0Z^3$, and where the energy of electron beta decay is much higher than the change in the total ionization energy of the atom involved, the change in the respective beta-decay endpoint energy is insignificant and does not therefore lead to a change in the decay probability. In those cases, the change in the probability of electron beta decay in an external magnetic field is due entirely to the change in the density of free electron states at the position of the nucleus [see Eq. (35)].

Thus, we have seen that the probability of the electron beta decay of a neutral atom and an ion in a superstrong constant uniform magnetic field increases.

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REFERENCES

1. A. G. Zhilich and B. S. Monozon, Fiz. Tverd. Tela **8**, 3559 (1966) [Sov. Phys. Solid State **8**, 2846 (1966)].
2. B. B. Kadomtsev, Zh. Éksp. Teor. Fiz. **58**, 1765 (1970) [Sov. Phys. JETP **58**, 945 (1970)].
3. B. B. Kadomtsev and V. S. Kudryavtsev, Pis'ma Zh. Éksp. Teor. Fiz. **13**, 61 (1971) [JETP Lett. **13**, 61 (1971)].
4. B. B. Kadomtsev and V. S. Kudryavtsev, Zh. Éksp. Teor. Fiz. **62**, 144 (1972) [Sov. Phys. JETP **35**, 76 (1972)].
5. L. A. Bureeva and V. S. Lisitsa, *Perturbed Atom* (IzdAT, Moscow, 1997), p. 189 [in Russian].
6. I. M. Ternov, V. N. Rodionov, V. G. Zhulego, and A. I. Studenikin, Yad. Fiz. **28**, 1454 (1978) [Sov. J. Nucl. Phys. **28**, 747 (1978)].
7. I. M. Ternov, V. N. Rodionov, and O. F. Dorofeev, Yad. Fiz. **37**, 875 (1983) [Sov. J. Nucl. Phys. **37**, 521 (1983)].
8. E. Kh. Akhmedov, Zh. Éksp. Teor. Fiz. **85**, 1521 (1983) [Sov. Phys. JETP **58**, 883 (1983)].
9. J. N. Bahcall, Phys. Rev. **124**, 495 (1961).
10. I. S. Batkin, Izv. Akad. Nauk SSSR, Ser. Fiz. **40**, 1279 (1976).
11. K. Takahashi and K. Yokoi, Nucl. Phys. A **404**, 578 (1983).
12. K. Takahashi, R. N. Boyd, G. J. Mathews and K. Yokoi, Phys. Rev. C **36**, 1522 (1987).
13. F. Bosch, T. Faestermann, J. Friese, et al., Phys. Rev. Lett. **77**, 5190 (1996).
14. V. A. Akulov and B. A. Mamyrin, Usp. Fiz. Nauk **173**, 1187 (2003) [Phys. Usp. **46**, 1153 (2003)].
15. A. A. Rukhadze, L. I. Urutskoev, and D. V. Filippov, Yad. Fiz. **69**, 820 (2006) [Phys. At. Nucl. **69**, 792 (2006)].
16. S. V. Starodubtsev and A. M. Romanov, *Radioactive Transformations of Nuclei and Atomic Shell* (Akad. Nauk UzSSR, Tashkent, 1958), p. 238 [in Russian].
17. L. I. Urutskoev and D. V. Filippov, Usp. Fiz. Nauk **174**, 1355 (2004) [Phys. Usp. **47**, 1257 (2004)].
18. B. S. Dzhelepov, L. N. Zyr'yanova, and Yu. P. Suslov, *Beta Processes* (Nauka, Leningrad, 1972) [in Russian].
19. V. P. Krainov, Zh. Éksp. Teor. Fiz. **64**, 800 (1973) [Sov. Phys. JETP **37**, 406 (1973)].
20. A. A. Sokolov and I. M. Ternov, *Relativistic Electrons* (Nauka, Moscow, 1974) [in Russian].
21. N. B. Delone and V. P. Krainov, *Atom in a Strong Light Field* (Énergoatomizdat, Moscow, 1984) [in Russian].
22. L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics,* Vol. 3: *Quantum Mechanics: Non-Relativistic Theory* (Fizmatlit, Moscow, 2001; Pergamon, New York, 1977).

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