

## EFFECTS OF ATOMIC ELECTRONS ON NUCLEAR STABILITY AND RADIOACTIVE DECAY

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### 1. Introduction

- (1) The only valid (necessary and sufficient) condition for  $\beta$ -stability of a *nucleus* is the minimum value of the mass of the *atom* (not of the nucleus) on the isobar line (i.e., atomic weight = const.). This condition holds true for all known isotopes without exception.
- (2) The *condition of nuclear stability* and the *decay schemes* of unstable nuclei depend on the state of the electron shells. Atom ionization and other perturbations in electron shells (caused, e.g., by magnetic fields) not only changes the decay periods of unstable nuclei,<sup>1,2</sup> but also alters decay schemes<sup>3</sup> and modifies the stability condition. For example, the  $^{163}\text{Dy}$ ,  $^{193}\text{Ir}$ ,  $^{205}\text{Tl}$  nuclei, which are stable in neutral atoms, become  $\beta$ -active when atoms are completely ionized. This means that by affecting electron shells one can alter conditions of nuclear  $\beta$ -stability and thus initiate nuclear transmutations by means of weak interactions.
- (3) We have developed a *phenomenological model* for the low-energy nuclear transmutation. According to the above observations, the probability of a nuclear reaction can significantly increase under the effects exerted on atomic electron shells. Without considering specific mechanisms of nuclear transformations, we have managed to determine the possible transformation products only using *basic conservation laws* (the energy and electric, baryon, and lepton charges). Amazingly, such a simple model yields results in qualitative agreement with experimental data.

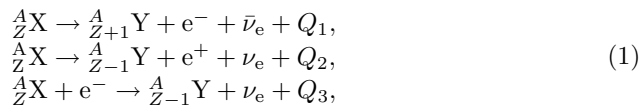
### 2. Condition of Nuclear $\beta$ -stability

The discussion of the conditions for  $\beta$ -stability of nuclei began at the very dawn of nuclear physics.<sup>4,5</sup> However, up to the 1950s, because there was not sufficient accuracy or experimental data on the masses of nuclear isotopes, it was not possible to assess the correspondence between theoretical forecasts and experimental data.

Inasmuch as at that time the accuracy of available experimental data did not always allow researchers to distinguish between nuclear masses and atomic masses, the conditions of the “minimum nuclear mass,” “minimum atomic mass,” and “maximum nuclear binding energy” on the isobar line seemed to be identical, while deviations from the assumed stability conditions were considered as exceptions.<sup>4,5</sup> Nowadays, thanks to available data,<sup>6</sup> it is possible to formulate (and check) an accurate condition for nuclear stability. The analysis of the database has shown that the stability conditions such as “minimum nuclear mass”<sup>7,8</sup> or “maximum binding energy”<sup>9</sup> on the isobar line are not accurate.

The *only condition* for  $\beta$ -stability of the *nucleus* in a neutral atom, which is *absolutely accurate*, is the requirement that the *atom mass* attains its minimum on the isobar.<sup>10,11</sup>

Let us discuss this statement in more detail. Consider the stability of a nucleus with respect to processes not involving a change in the number of nucleons in the nucleus, that is, those occurring due to weak interactions, namely, the electron ( $\beta^-$ ) or positron ( $\beta^+$ )  $\beta$ -decay and the K-capture:



where  $\nu_e$  and  $\bar{\nu}_e$  are the electronic neutrino and antineutrino, respectively, and X and Y are the nuclei with the atomic weight  $A$  and charge  $Z$  (in the electron charge units).

It is well known<sup>10,11</sup> that the energy produced ( $Q > 0$ ) or absorbed ( $Q < 0$ ) in such nuclear reactions can be determined using the mass difference between the initial nuclei and the reaction products:

$$Q = M_N(A_X, Z_X) - M_N(A_Y, Z_Y) \mp m_e, \quad (2)$$

where  $M_N(A, Z)$  is the mass of the nucleus  ${}^A_Z\text{X}$ ,  $m_e$  is the electron rest mass; a “−” corresponds to  $\beta^\pm$ -decay ( $Q_1$  and  $Q_2$ ), and a “+” to the K-capture ( $Q_3$ ). As the K-capture always involves less energy than the positron  $\beta^+$ -decay ( $Q_3 - Q_2 = 2m_e$ ), the possibility of a positron  $\beta^+$ -decay does not alter the nuclear stability condition.

$$M_N(A, Z) = (A - Z)m_n + Zm_p - W_N(A, Z), \quad (3)$$

where  $m_p$  and  $m_n$  are the rest masses of the proton and neutron, respectively. The binding energy  $W_N$  is the energy that is to be “pumped” to the nucleus to separate it into constituent nucleons. Expression (2) is valid in the case where the nucleus has no electron shell. When a nucleus decays as a part of a neutral atom, the binding energy of electrons is to be taken into account. Since the first ionization potential is not over 25 eV, this energy can be neglected when compared to the accuracy to which the nuclear binding energy is measured ( $\sim 1$  keV). In this approximation, the

energy produced in the K-capture and electron  $\beta^-$ -decay of a neutral atom equals:

$$Q = M_A(A_X, Z_X) - M_A(A_Y, Z_Y), \quad (4)$$

where

$$M_A(A, Z) = (A - Z) m_n + Z (m_p + m_e) - W(A, Z) \quad (5)$$

is the mass of the *atom* and  $W$  is the binding energy of the nucleus in the atom taking into account the *full atom ionization energy*  $I(Z)$ :

$$W(A, Z) = W_N(A, Z) + I(Z). \quad (6)$$

This is the energy needed to separate a neutral atom into the constituent protons, neutrons, and electrons.

With an accuracy of  $Z I_H$  ( $I_H = 13.6 \text{ eV}$  is the hydrogen ionization potential), which for  $Z < 100$  is not worse than the accuracy to which the nuclear binding energy is measured, the energy defined in this way is the same as the energy needed to separate the nucleus into neutrons and hydrogen atoms:

$$M_A(A, Z) = (A - Z) m_n + Z M_H - W(A, Z), \quad (7)$$

where  $M_H$  is the mass of the hydrogen atom. Historically, the nuclear binding energy was introduced for calculating the energy produced in nuclear reactions involving neutral atoms. Therefore, tables<sup>6</sup> quote these atomic energies  $W$ , which include the full ionization potential  $I(Z)$ , rather than the nuclear  $W_N$  values. For determining the energy of the nucleus, one can also use the mass defect  $\Delta M$ , which is related to  $M_A$  as follows<sup>6</sup>:

$$M_A(A, Z) = A m_{\text{a.e.m.}} + \Delta M(A, Z), \quad (8)$$

where  $m_{\text{a.e.m.}} \approx 931.5 \text{ MeV}$  is the atomic mass unit; for the mass defect, the normalization  $\Delta M(^{12}\text{C}) = 0$  was chosen.

It is well known<sup>10,11</sup> that for a nucleus to be  $\beta$ -stable, it is *sufficient* that all possible decay channels be closed for energy reasons ( $Q < 0$ ). Hence, the sufficient condition for  $\beta$ -stability of a nucleus can be formulated as the *minimum atomic mass*  $M_A(A, Z)$  [which is equivalent to the minimum mass defect  $\Delta M(A, Z)$ ], including all local minima on the isobar ( $A = \text{const.}$ ).

Note that this requirement involves the minimum *atomic mass*  $M_A(Z)$  and not the minimum *nuclear mass*  $M_N(Z)$  or the maximum binding energy  $W(Z)$  (these conditions are not the same!). The functions  $M_A(Z)$ ,  $M_N(Z)$ , and  $W(Z)$  are related to each other in the following way:

$$\begin{aligned} M_N(Z) &= M_A(Z) + I(Z) - Z \cdot m_e, \\ -W(Z) &= M_A(Z) - A \cdot m_n + Z \cdot \tilde{m}, \end{aligned} \quad (9)$$

where  $\tilde{m} = m_n - m_p - m_e = 782.3 \text{ keV}$ .

Since the functions  $M_N(Z)$  and  $W(Z)$  differ from  $M_A(Z)$  on the isobars ( $A = \text{const.}$ ) by terms monotonic in  $Z$  (9), then, in qualitative terms, all of these three functions ( $M_A$ ,  $M_N$ ,  $W$ ) feature the same behavior, but the minima of  $M_N(Z)$  may

be shifted to larger  $Z$  and the binding energy  $W(Z)$  maxima may be shifted to smaller  $Z$  with respect to the minima of the  $M_A(Z)$  function [the latter are the same as the minima of  $\Delta M(Z)$ ].

The dependence of the binding energy on the nuclear charge can be qualitatively described by the well-known semi-empirical Weizsäcker formula:<sup>11</sup>

$$M_A(A, Z) = A m_n - Z \tilde{m} - a_V A + a_S A^{2/3} + a_C \frac{Z(Z-1)}{A^{1/3}} + a_{\text{SYM}} \frac{\left(\frac{A}{2} - Z\right)^2}{A} - a_P \frac{\delta}{A^P} - I(Z), \quad (10)$$

where  $a_V = 15.75 \text{ MeV}$ ,  $a_S = 17.8 \text{ MeV}$ ,  $a_C = 0.71 \text{ MeV}$ ,  $a_{\text{SYM}} = 94.8 \text{ MeV}$  and  $a_P = 34 \text{ MeV}$  are the coefficients of the nuclear energy: the volume, surface, Coulomb, symmetry, and coupling energy, respectively. The coefficient  $\delta$  describes the coupling effect:  $\delta = 0$  for the nuclei with odd  $A$ ,  $\delta = 1$  for even-even nuclei (an even number of neutrons and an even number of protons), and  $\delta = -1$  for odd-odd nuclei. The exponent  $P$  in the last term (coupling) varies, depending on the author, from  $1/3$  to  $1$ .

Recall the well-known fact which follows from the Weizsäcker formula (10): on isobars with odd  $A$ ,  $M_A(Z)$  is described by a parabola with one minimum ( $\delta = 0$ ) (Fig. 1a); on isobars with even  $A$ ,  $M_A(Z)$  is described by a broken line confined by two parabolas which correspond to even  $Z$  ( $\delta > 0$ ) and to odd  $Z$  ( $\delta < 0$ ) (Fig. 1b, c). In the latter case, the function  $M_A(Z)$  can develop (depending on  $A$ ) one, two, or three minima. Figure 1b illustrates the case when, for even  $A$ , the parabola minimum corresponds to even  $Z$ , and Fig. 1c shows the parabola minimum corresponding to odd  $Z$ .

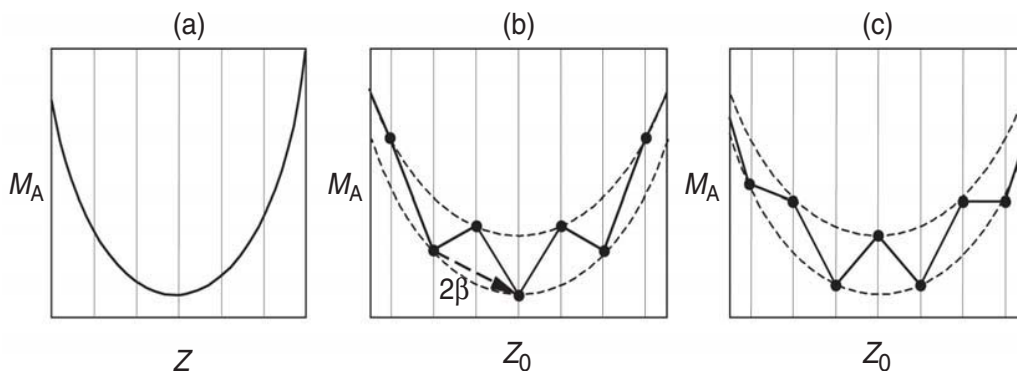


Figure 1. Dependence of the atomic mass on the charge.  $Z_0$  is the parabola minimum. (a) For odd atomic mass  $A$ , (b) for even  $A$  and even  $Z_0$ , and (c) for even  $A$  and odd  $Z_0$ .

A straightforward analysis of the database<sup>6</sup> shows that *all stable isotopes without an exception* correspond to *minima of atomic masses  $M_A(Z)$  on the isobars*.

Moreover, all  $\beta$ -decay and K-capture processes that are *energetically allowed* are *realized* in nature (no other prohibitions are involved). Thus the following statement is correct.

For a *nucleus* in a neutral atom to be  $\beta$ -stable (stable with respect to single  $\beta^\pm$ -decay and K-capture processes), it is *necessary and sufficient* that this isotope correspond to a *minimum of the atomic mass* on the isobar ( $A = \text{const.}$ ).

Note that all 12 natural isotopes that do not correspond to a minimum of  $M_A(Z)$  are unstable. Even though they are long lived, nevertheless they are not stable (Table 1). On the other hand, there are no natural  $\beta$ -stable isotopes with atomic

Table 1. Unstable natural isotopes.

Isotope	Natural abundance (%)	Channels of nuclear decay	Energy of nuclear transition (keV)	Half life (years)
$^{40}\text{K}$	0.012	$\beta^-$ 89.3 $\epsilon(\beta^+)$ 10.7	1311 1505	$1.277 \times 10^9$
$^{48}\text{Ca}$	0.187	$\beta^-$	278	$6 \times 10^{18}$
$^{50}\text{V}$	0.25	$\beta^-$ 17 $\epsilon(\beta^+)$ 83	1037 2208	$1.4 \times 10^{17}$
$^{87}\text{Rb}$	27.85	$\beta^-$	283	$4.75 \times 10^{10}$
$^{96}\text{Zr}$	2.8	$\beta^-$		$3.8 \times 10^{19}$
$^{113}\text{Cd}$	12.22	$\beta^-$	316	$7.7 \times 10^{15}$
$^{115}\text{In}$	95.77	$\beta^-$	496	$4.4 \times 10^{14}$
$^{123}\text{Te}$	0.9	$\epsilon$	53	$> 10^{13}$
$^{138}\text{La}$	0.09	$\beta^-$ 33.6 $\epsilon(\beta^+)$ 66.4	1044 1738	$1.05 \times 10^{11}$
$^{176}\text{Lu}$	2.59	$\beta^-$	1192	$3.78 \times 10^{10}$
$^{187}\text{Re}$	62.6	$\beta^-$	2.66	$4.35 \times 10^{10}$
$^{180}\text{Ta}^m$	0.012	$\gamma$	75.3	$1.2 \times 10^{15}$

mass 5 or 8, as they are unstable with respect to decay:  $^5\text{He} \rightarrow ^4\text{He} + n$ ,  $^8\text{Be} \rightarrow 2^4\text{He}$ . For atomic masses  $A > 141$ ,  $\alpha$ -decay becomes energetically allowed, while for some isotopes with atomic masses in the  $210 > A > 141$  range, it is forbidden, and all isotopes with  $A > 209$  are  $\alpha$ -active. The isotope  $^{180}\text{Ta}^m$ , which is observed in nature, is a long-lived ( $1.2 \times 10^{15}$  years) isomeric excited state of the nucleus. The very long half-life time is due to the large difference of spins in the isomeric ( $9^-$ ) and ground ( $1^+$ ) states.

The analysis of the database shows that the assumption that the minimum nuclear mass  $M_N(Z)$  as a sufficient condition for  $\beta$ -stability is not accurate. For example, more than 30 isotopes that correspond to minima of the nuclear mass  $M_N(Z)$  on the isobars are unstable with respect to the K-capture.

The following example describes a typical situation: the minimum atomic mass for the isobar with the atomic weight 55 is attained on only one stable manganese isotope  $^{55}\text{Mn}$ , while the nuclear mass attains its minimum on the unstable isotope  $^{55}\text{Fe}$  (the decay period is 2.7 years).

The  $^{55}\text{Mn}$  nucleus is *heavier* than the  $^{55}\text{Fe}$  nucleus:

$$M_N(^{55}\text{Mn}) - M_N(^{55}\text{Fe}) \approx 280 \text{ keV},$$

whereas the  $^{55}\text{Mn}$  atom *is lighter* than the  $^{55}\text{Fe}$  atom:

$$M_{\text{A}}(^{55}\text{Fe}) - M_{\text{A}}(^{55}\text{Mn}) \approx 231 \text{ keV}.$$

### 3. The $\beta$ -stability of Ionized Atoms

Further, we discuss how the changes in electron shells, that is, changes whose energies are much *smaller* than nuclear binding energies, can cause modifications to the nuclear stability condition. To determine the relation between the charge  $Z$  and the mass  $A$  of a stable isotope, let us find the minimum of the atomic mass  $M_{\text{A}}(Z)$  on the isobar. The atomic mass can be represented in the form:

$$M_{\text{A}}(A, Z) = C_1(A) + C_2(A) (Z - Z_0)^2 - \delta(A, Z) a_{\text{P}} A^{-P}, \quad (11)$$

where

$$\begin{aligned} Z_0 &= \frac{A a_{\text{SYM}} + a_{\text{C}} A^{-1/3} + \tilde{m}}{2 a_{\text{SYM}} + a_{\text{C}} A^{2/3}}, \\ C_2(A) &= \frac{a_{\text{SYM}}}{A} + a_{\text{C}} A^{-1/3}, \\ C_1(A) &= A(m_{\text{n}} - a_{\text{V}}) + a_{\text{S}} A^{2/3} - Z_0^2 C_2(A) + a_{\text{SYM}} \frac{A}{4}. \end{aligned} \quad (12)$$

Inasmuch as  $Z$  can only be integer, the minimum of  $M_{\text{A}}(Z)$  is attained at an integer  $Z$  nearest to  $Z_0$ .

When the ionization potential of an electron shell changes (due to, e.g., the effect of a strong magnetic field), the stability condition has a similar form, but involves the following replacement  $\tilde{m} \rightarrow \tilde{m} + O(m_{\text{e}})$ .

With  $O(m_{\text{e}}) \ll a_{\text{SYM}} = 94.8 \text{ MeV}$ , the difference between these two conditions seems to be negligible. However, in the cases when  $Z_0$  is close to a half-integer value, the perturbation, even as small as  $m_{\text{e}}/a_{\text{SYM}}$ , can change the integer nearest to  $Z_0$  by unity. This means that it is the nucleus of the neighboring isotope that will become stable.

Using the Thomas–Fermi model for the atom ionization potential:<sup>12</sup>

$$I(Z) \cong 20.8 Z^{7/3} \text{ eV} \quad (13)$$

and the formula for the ionization potential of hydrogen-like atoms (consisting of a nucleus and a single electron):<sup>12</sup>

$$I^{1\text{e}}(Z) = 13.6 Z^2 \text{ eV}, \quad (14)$$

we deduce that the difference between ionization potentials of the neighboring elements

$$I(Z+1) - I(Z) \propto Z^{4/3} \quad (15)$$

increases slower than the ionization potential of the hydrogen-like atom and that basically for all atoms ( $Z > 7$ ):

$$I(Z+1) - I(Z) < I^{1\text{e}}(Z) < I^{1\text{e}}(Z+1). \quad (16)$$

Hence, the energy of the  $\beta^-$ -decay of a fully ionized nucleus to a bound state of the electron equals:

$$Q = Q_0 + I(Z) - I(Z + 1) + I^{1e}(Z + 1) > Q_0, \quad (17)$$

where  $Q_0$  is the decay energy of the nucleus in the neutral atom. This means that when the nucleus is fully ionized, the  $\beta^-$ -decay to the bound state always yields more energy than the  $\beta^-$ -decay of the neutral atom.

The analysis of the database shows that some stable nuclei of neutral atoms

$$^{163}\text{Dy}, \quad ^{193}\text{Ir}, \quad ^{205}\text{Tl}$$

become unstable with respect to the  $\beta^-$ -decay to a bound state when they are fully ionized. This effect has been observed experimentally.<sup>1</sup>

When the atom is completely ionized, the nuclear stability condition always shifts to larger  $Z$ . For nuclei emitting delayed neutrons this results, in particular, in the increase of the fraction of delayed neutrons, as the nuclei become more neutron-redundant upon ionization.

#### 4. A Phenomenological Model of Nuclear Transformation

We have shown that the conditions can be created (e.g., by application of a strong magnetic field) when the distortion of electron shells would result in a significant increase in the probability of nuclear processes involving weak interactions.

Therefore, we consider the question whether the isotopic composition currently observed in nature is final and equilibrium. The changes in the isotopic composition of elements toward  $^{56}\text{Fe}$ , due to fission and fusion reactions accompanied by great energy changes is known to be energetically favorable.

Below we show that within a small (on the nuclear scale) energetic interval, a plethora of states can exist which are populated in nature in a very uneven way. Since the nuclear states being considered basically have the same energies (to within the accuracy of measurements), one nuclear state can transmute to another in a resonant transition without emission of a significant amount of energy (in the nuclear scale), which is without radioactivity. In standard nuclear reactions, the final nuclei are usually produced in excited states as a result of collisions between high-energy nuclei. Because of that usual nuclear reactions feature radioactivity phenomena. We consider non-standard nuclear transitions between initial and final states, which have the same nuclear energies. It should be stressed that we only discuss the question whether low-energy transitions are allowed by conservation laws, leaving aside the mechanisms underlying such transitions.

Consider hypothetical collective nuclear transformation processes involving weak interactions, which satisfy the following equation:

$$\sum_i \frac{A_i}{Z_i} X_i \rightarrow \sum_j \frac{A_j}{Z_j} Y_j + k e^- + k \bar{\nu}_e + Q, \quad (18)$$

where  $X_i$  and  $Y_j$  are the nuclei with the atomic weight  $A_i$  and charge  $Z_i$ , respectively, a special case of which are neutrons ( $A = 1$ ,  $Z = 0$ );  $k$  is the number of the electrons

involved in the reaction;  $k$  can be positive (in  $\beta^-$ -decay), negative (in K-capture), or equal to zero (in the case of strong interactions alone). Let us assume that generically some of the  $X_i$  (or  $Y_i$ ) nuclei may be identical.

Define a set  $\mathfrak{R}$  of nuclear ensembles  $\{X_i\}$ , that is, an ensemble of nuclei is an element of  $\mathfrak{R}$ . Then transformation is a transition between two elements of the set  $\mathfrak{R}$ . The development of the phenomenological model for such transformation reduces then to the following:

in the set  $\mathfrak{R}$  of the ensembles of nuclei, it is necessary to find the ensembles  $\{Y_i\}$ , the energies of which are closest to those of the initial ensemble  $\{X_i\}$  provided that the baryon charge (the number of nucleons), the electric charge and the lepton charge are conserved.

Recall that neutrinos take away part of the released energy  $Q$ . However, the energy scale in the case being considered is significantly smaller than in usual non-collective reactions involving weak interactions: in the hypothetical collective process, a neutrino may have any small momentum. The first attempt to develop such phenomenological model of nuclear transformations was made by Russian researchers in Dubna (Kuznetsov's group).<sup>13</sup>

Each ensemble of nuclei  $\{X_i\}$  is described by a set of integers  $\{a_i\} \equiv \vec{a}$ , where  $a_i$  is the number of nuclei  $X_i$  in the ensemble; the unit vector  $\vec{\delta}_i$  corresponds to an ensemble comprising only one nucleus  $X_i$ .

In the set  $\mathfrak{R}$  we define a norm:

$$\|\vec{a}\| \equiv \sum_i a_i A_i q_i,$$

$$q_i \equiv q(X_i) \equiv \Delta M({}^1_1\text{H}) - \frac{\Delta M(X_i)}{A_i} = \frac{W}{A_i} - \left(1 - \frac{Z}{A_i}\right) \tilde{m}, \quad (19)$$

where  $W$  is the binding energy of the nucleus and  $Z$  is its charge  $\tilde{m} = m_n - m_p - m_e = 782.3 \text{ keV}$ . Using the data on isotopes, one can see readily that  $q(X) > 0$  for all known isotopes (including unstable ones), and the maximum of  $q(X)$  is attained for  ${}^{56}\text{Fe}$ .

The introduced norm has the following physical meaning: it equals the energy needed to separate the atom into hydrogen atoms, i.e. to separate the nucleus into the constituent protons and neutrons with the subsequent transformation of all neutrons into protons:

$$X_i \rightarrow A_i {}^1_1\text{H} + (A_i - Z_i) e^- + (A_i - Z_i) \bar{\nu}_e - Q, \quad Q = \|\vec{\delta}_i\|.$$

Actually, the ensemble consisting of hydrogen atoms is used as the origin (zero norm).

When comparing the binding energies  $W$  of two nuclei,  ${}^3\text{He}$  and  ${}^3\text{H}$  (tritium), we determine that the binding energy of the stable nucleus  ${}^3\text{He}$  is *lower* than that of the unstable one  ${}^3\text{H}$ .



This apparent contradiction (larger binding energy is expected to correspond to a more stable nucleus) is resolved when the considered norm is used:

In this formulation, the problem of modelling the transformation process is reduced to a search for ensembles  $\{b_i\} \equiv \vec{b}$ , whose norm is close to the initial one  $\{a_i\} \equiv \vec{a}$ , i.e.:  $\left| \|\vec{b}\| - \|\vec{a}\| \right| < \varepsilon$ ,  $\varepsilon$  being small. One can check easily that the transition from the ensemble  $\vec{a}$  to the ensemble  $\vec{b}$  corresponds to a transformation of type of the following form:

$$\sum_i a_i \frac{A_i}{Z_i} X_i \rightarrow \sum_i b_i \frac{A_i}{Z_i} X_i + N_H {}^1_1\text{H} + k e^- + k \bar{\nu}_e + Q, \quad (20)$$

where  $N_H = \sum_i (a_i - b_i) A_i$  is the number of produced ( $N_H > 0$ ) or absorbed ( $N_H < 0$ ) protons,  $k = \sum_i (a_i - b_i) (A_i - Z_i)$  is the number of produced ( $k > 0$ ) or absorbed ( $k < 0$ ) electrons, and  $Q = \sum_i (b_i - a_i) A_i q_i$  is the produced ( $Q > 0$ ) or absorbed ( $Q < 0$ ) energy.

The use of the proposed norm makes it possible to significantly simplify the task of selecting the nuclear ensembles, as actually, when selecting  $\vec{b}$ , it is not necessary to check that the electric and baryon charge conservation laws are observed as these conservation laws are always observed owing to the specified conditions.

The search for the nearest ensembles is reduced to an algebraic problem. Note that if one assumes that the binding energies and hence norms  $Q_1 = \|\vec{\delta}_1\|$  and  $Q_2 = \|\vec{\delta}_2\|$  of two nuclei  $X_1$  and  $X_2$  are exactly known and can be expressed as rational numbers, then the ratio  $Q_1/Q_2$  can be expressed as a ratio of two integers  $N_2/N_1$ . One can check easily that in this case two ensembles  $N_1 X_1$  and  $N_2 X_2$  have *equal* energies. This implies that the transition  $N_1 X_1 \rightarrow N_2 X_2 + \dots$  can occur without changes in energy. However, the coefficients obtained in this case (the number of particles in each ensemble) can be so large that such a transformation will be of no practical interest. The quoted example only illustrates the theoretical possibility to find ensembles, which have very close energy values. In practice, the numerical selection of the ensembles is restricted by the accuracy to which the nuclear binding energy is measured ( $\Delta Q \sim 1$  keV).

Taking into account huge nuclear binding energies, the discrete nature and the finite size of Mendeleev's table, it is not a priori obvious that it is possible to find different nuclear ensembles with the same number of nucleons and with nuclear energies differing by values of the order of the electron chemical binding energy. However, this is what we have succeeded in doing.

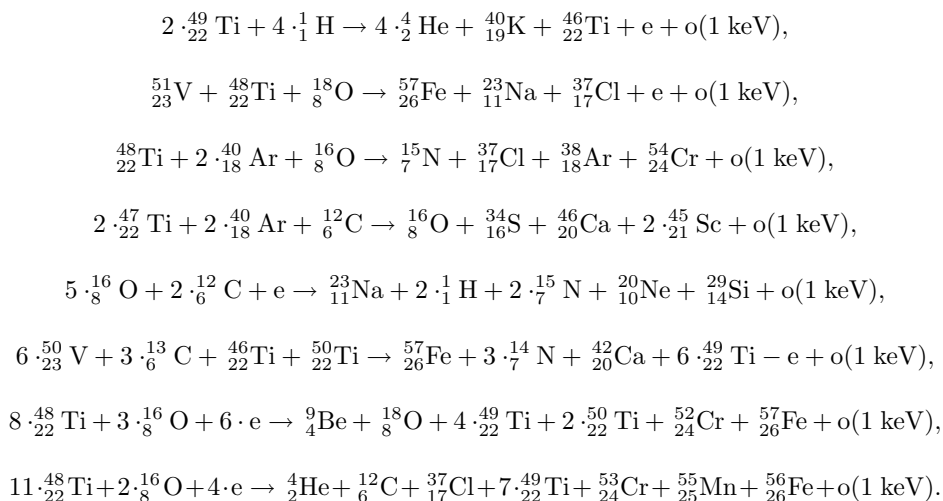
When developing the transformation model, the initial nuclear ensemble is divided into groups called "clusters," and the search for ensembles with the nearest energy values is carried out separately for each cluster. The calculation model contains a number of parameters: the range of considered energy changes in the cluster, the cluster's size, and the number of the nucleons transferred between the cluster's nuclei.

The number of ways in which an initial cluster containing  $N$  nucleons can be "reshuffled" is  $\sim 2^N$ . Thus, it is so large that it is impossible to consider all con-

figurations. It seems to be reasonable to assume that the number of nucleons in a cluster is limited by the closest geometrical neighbors, that is, it is not over 20. This approach makes it possible to significantly reduce the number of combinations involved in processing. The final calculation result is obtained by averaging all possible simplest transformations in clusters taking into account the statistical weight of each cluster in accordance with the initial distribution of nuclei. The final nuclear distribution shows what nuclei have appeared and what disappeared in the process of transformation.

Note that the minimum number of cluster nuclei, for which a non-trivial solution can be found, equals to three.

In order to determine the parameters of the phenomenological model that correspond to experimental data, we have modelled the transformation of titanium foil in water and glycerol in an argon atmosphere. In this case, the ensembles with the minimum energy differences  $|Q| < 1$  keV (with energy changes smaller than the accuracy of the binding energy measurement) were as follows:



Note that the nuclear energies of the left-hand side and right-hand side nuclear ensembles are the same (to within the accuracy to which the binding energy is measured). This means that the energy changes in such nuclear transformations are of the order of chemical energies.

It should be stressed that when solving such a problem, a large number of combinations ( $10^5$ – $10^6$ ) is processed, and the examples quoted above are nothing but *illustrations*. It would be incorrect to consider the quoted combinations as nuclear reactions where a large number of nuclei collide. These examples describe transitions between states of nuclei, the mechanism of such transition being obscure as yet. One can only assume that some transition (possibly a resonant one) occurs due to effects of a new interaction.

We have shown that there are nuclear ensembles consisting of stable nuclei, which have identical electric and baryon charges and the energy values that are

very close to each other. Hence, transitions between such ensembles do not violate the known conservation laws and are not accompanied by radioactive emission of energy. We do not consider the transformation mechanisms, but note that in the resonance conditions, the probability of penetrating through a potential barrier does not depend on its height or width.

This model needs detailed investigation by comparing experimental results with the results of modelling for different parameters. Such studies are currently in progress, but first it was necessary to find a qualitative effect predicted by the model, which could be checked experimentally.

The numerical simulation of combinations, we have carried out, shows that when a sufficient amount of vanadium is transformed, the isotopic composition of iron should be distorted and shifted toward an increase in the  $^{57}\text{Fe}$  isotope content. This feature is specific to vanadium and the result does not depend on the calculation parameters: the quantitative ratio of iron isotopes changed, but the amount of  $^{57}\text{Fe}$  was always significantly larger than that in the natural mixture of iron isotopes. The proposed phenomenological model cannot make quantitative predictions for the content of  $^{57}\text{Fe}$ . However, it predicts qualitatively an increase in the  $^{57}\text{Fe}$  content as compared to that in the natural mixture of isotopes (2.2%), due to the transformation of V. The experiment, in which titanium foil was exploded using the electric charge in solutions of vanadium salts ( $\text{VCl}_3$  and  $\text{NH}_4\text{VO}_3$ ), has shown that the content of  $^{57}\text{Fe}$  in Fe shifts to larger values (to  $3.7 \pm 0.5\%$ ).

We have shown that even without developing a specific model for low-energy nuclear transformation but using conservation laws alone, one can find examples of nuclear ensembles that differ only by chemical scale energies.

## 5. Conclusions

- (1) The only valid condition of  $\beta$ -stability of a *nucleus* is the minimum value of the mass of the *atom* on the isobar.
- (2) The *condition of nuclear stability* and the *decay schemes* of unstable nuclei depend on the state of the electron shells. Atom ionization and other perturbations in electron shells (caused for example by magnetic field)
  - not only change the decay periods of unstable nuclei,
  - but also alter decay schemes
  - and modify the stability condition.
- (3) We have developed a *phenomenological model* for the low-energy nuclear transmutation.

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